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MALTHUSIAN DYNAMICS AND THE RISE OF THE POOR MEGACITY

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ABSTRACT

The largest cities in the world today lie mainly in relatively poor countries, which is a departure from historical experience, when the largest cities were typically found in the richest places. Further, these poor mega-cities have grown through high rates of urban natural increase, as opposed to the in-migration that drove historical urbanization. In this paper we provide an explanation for the rise of these poor mega-cities and their departure from the historical norms. Combining models of urban agglomeration and congestion with Malthusian models of endogenous population, we show that cities can exert Malthusian forces on living standards. Poor mega-cities have grown because their rapid rate of natural increase exerts a downward force on urban wages, which ensures that natural increase remains high, the mega-cities remain poor, and their size is unrelated to productivity. In comparison, richmega-cities of the past had low rates of natural increase, ensuring that they could take advantage of agglomeration technologies, and city growth occurred in response to productivity improvements. Our work shows that Malthusian forces do not disappear just because an economy urbanizes, and that urban Malthusianism combines with rapid urban population growth to drive developing poor mega-cities towards stagnation even if they do not face limits to their resource stocks.

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1. INTRODUCTION

Urbanization has gone hand in hand with economic growth throughout history, and the majority of world population now lives in cities. However, post-war developing countries have urbanized in a fundamentally different manner compared to the historical experience of developed countries. Specifically, the post-war period has seen the rise of poor mega-cities in developing nations, which we document in detail below. Delhi, Dhaka, Kinshasa, Karachi, and Lagos are some of the very largest urban agglomerations on the planet today, each with over 10 million inhabitants. Only six of the currently largest 30 cities (Tokyo, New York, Los Angeles, Paris, Osaka, and London) today are in high income countries. The prevalence of poor mega-cities today runs counter to historical experience. In the first half of the 20th century, the very largest urban agglomerations in the world were all in the most advanced economies (e.g. New York, London, Paris, Berlin, Chicago) with only a few major cities in poor countries, like Mumbai, reaching even one-fourth of their size.

The mega-cities of today's developing world are also unlike their historical peers in that their massive size is *not* indicative of rapid economic growth or higher living standards. For the poorest countries, urbanization rates in 2010 are roughly 10 percentage points higher than in 1950, and 20 percentage points higher than they were in 1910 for identical levels of GDP per capita. Developing countries of today are urbanizing into *poor* mega-cities that do not appear to be able to take advantage of the externalities or agglomeration economies of their rich-country peers.

Our aim in this paper is to provide an explanation for the rise of these poor mega-cities, and why they differ from the historical experience of urbanization and rapid economic growth. We propose that these mega-cities grew in poverty - and tend to remain mired in poverty - because of what we characterize as urban Malthusian forces. To describe these forces, we build a model that combines the insights of Malthusian models of endogenous population growth (Ashraf & Galor, 2011; Galor & Weil, 2000) with the urban literature on equilibrium city size (Duranton & Puga, 2004; Duranton, 2013). The Malthusian literature describes how limited resources will tend to restrict living standards to some subsistence level under the assumption that population growth responds positively to living standards. The urban literature captures the tension between the positive effects of agglomeration economies and the negative effects of congestion to find that urban wages display an inverted-U shape with respect to city size. For sufficiently large cities, the urban congestion effect dominates and this generates the typical Malthusian relationship of population and living standards.

Urban agglomerations thus can become "too big", at which point they become subject to the same type of Malthusian pressures that arise in models featuring fixed stocks of resources. Here, rather than limited agricultural land, the Malthusian pressures arise because urban congestion overwhelms any positive agglomeration economies. Sufficiently rapid city growth can drive *down* living standards.

Within this model, we are able to distinguish a poor mega-city equilibrium that differs from the historical equilibrium. Poor mega-cities arise and persist because their rates of natural increase are very high, which leads to large absolute size but also keeps living standards low through the urban Malthusian effects. Low living standards ensure that rates of natural increase remain high, and hence the poor mega-cities end up growing without bound. In particular, their growth is unrelated to productivity changes in the formal urban sector (which has positive agglomeration economies), as their size ensures that this technology is no longer viable due to congestion effects. In comparison, the historical equilibrium involves cities with low rates of natural increase, which allows them to maintain relatively high wages, which in turn reinforces the low rate of natural increase. These cities continue to use the formal urban technology, and grow in size precisely because of productivity increases with that technology.

This poor mega-city equilibrium differs from the historic experience because of their urban rate of natural increase is so large. We document more fully below that for modern developing nations, cities have rates of natural increase that are well above those seen historically. In particular, city mortality rates are quite low in developing countries today, well below levels seen at comparable periods of development for rich nations. These low mortality rates arose in the post-war period due to the epidemiological transition, which brought crude death rates in these cities down close to rich country levels. Crude birth rates in the poor mega-cities remained large, and similar to historical rates for large cities. The combination of the birth rates with the very low mortality rates within poor mega-cities implied a strongly positive urban rate of natural increase in these developing nations. In terms of our model, this high observed rate of natural increase meant rapid internal city growth, and pushed these cities into the poor mega-city equilibrium.

Because of the Malthusian nature of urban areas, our model has similar implications to typical Malthusian models. Lower congestion costs act like an expansion of the resource base in standard Malthusian models, expanding the size of cities before eventually returning to the subsistence living standard. Moreover, lower mortality rates and/or higher fertility rates are "bad" in this model, because they push down the subsistence living standard. The poor mega-city equilibrium is one that arises perversely because of the success of interventions that limited urban mortality rates while urban fertility remains relatively high. In this, our work is similar to others that emphasize the negative effect of mortality interventions and/or the positive impacts of mortality increases (Acemoglu & Johnson, 2007; Young, 2005).

The origin of poor mega-cities is certainly not mono-causal, and our explanation involving Malthusian effects should be thought of as a complement to those involving changes in urban transportation and housing technologies (i.e. the ability to build out or build up), changes in preferences towards urban areas, urban-biased policies, or others. One advantage of our explanation is that it is generalized, in the sense that all urban areas are likely to have the agglomeration and congestion effects driving our results. Other explanations would exacerbate or mitigate the effects we are discussing here, but do not rule out ours.

The main contribution of our paper is to explain the conditions under which these poor mega-cities will arise, and to describe what their future may look like. In addition to this, our work adds to the literature on the effects of demography on economic growth in general. Population growth promotes economic growth if high population densities encourage human capital accumulation or technological progress (Kremer, 1993; Becker, Glaeser & Murphy, 1999; Lagerlof, 2003). In the urban setting we consider, there are positive impacts of population growth due to agglomeration effects - but only over some range of population size. Population growth also induces congestion effects, which negatively affects living standards, similar to unified growth models that have a fixed resource (i.e. agricultural land) as part of the production process (Galor & Weil, 1999, 2000; Strulik & Weisdorf, 2008; Galor, 2011). Our work shows that the negative Malthusian effects need not arise because of natural resource limits, but rather due to the nature of urbanization. Significantly, this implies that Malthusian forces need not disappear as economies develop. Those forces remain operational even though agriculture and/or resource extraction cease to be significant sectors of production.

This paper is close in subject matter to Voigtländer & Voth (2013*a*), who study the possibility of multiple equilibria based on the urbanization process in historical Europe and China, but do not consider the modern arrival of poor mega-cities in the developing world. In terms of mechanisms, they introduce a non-linearity in mortality rates tied to a threshold urbanization *rate*, and thus an exogenous shock like the Black Death leads allows Europe to move to a better equilibrium. In comparison, our model introduces an explicit Malthusian force in city *size*, and the equilibrium that a city ends up is not a result of an exogenous shock to city size.

Our work also is linked to studies that look at the effect of mortality changes on population and economic growth (Acemoglu & Johnson, 2007; Bleakley, 2007; Bleakley & Lange, 2009; Bleakley, 2010; Cutler et al., 2010). Other studies have looked at the effects of unexpected decreases in population on development (Young, 2005; Voigtländer & Voth, 2009; Ashraf, Weil & Wilde, 2011; Voigtländer & Voth, 2013*a*,*b*). Relative to those works, we consider the implications of unexpected population increases, specifically from the perspective of cities. The few papers that have looked at cities examined how urbanization *rates* affected the demographic transition and developemtn (Sato & Yamamoto, 2005; Sato, 2007; Voigtländer & Voth, 2013*b*). In contrast, our model of urban Malthusianism highlights the fact that the absolute *size* of urban agglomerations is a crucial component of developement.

Glaeser (2013) begins with the same set of motivating facts regarding poor mega-cities, but looks at them in a different light. His paper focuses on the internal institutional structure that prevents mega-cities from reducing the overwhelming congestion effects of their size, examining the trade-off between disorder and dictatorship. His model does not consider the unique demographics leading to the origin of these mega-cities.

In the following section we document more fully the rise of poor mega-cities in the developing world, and the underlying demographic features of these cities. Then, with those stylized facts in place, we present our model of urban Malthusian dynamics, showing how an economy may find itself in an equilibrium with poor mega-cities. We then compare this equilibrium to the situation seen in historical European development.

2. FACTS ON THE RISE OF POOR MEGA-CITIES

Table 1 shows the largest 30 cities in select years from 1700 to 2015 (Chandler, 1987; United Nations, 2014). In 1700, the largest cities were under 1 million persons in size, with Istanbul, Tokyo, and Beijing all at around 700,000 inhabitants. While small in absolute size, the largest cities in 1700 were generally located in the most economically advanced areas of the world in that period. While London and Amsterdam had wages that were high relative to other European cities and the rest of the world, cities such as Beijing, Tokyo, and Istanbul all had wages equivalent to those found in cities such as Paris and Naples (Allen, 2001; Allen et al., 2011; Özmucur & Pamuk, 2002).

By 1900, the nature of the list of largest cities changed along several dimensions. First, the absolute sizes were roughly an order of magnitude larger than in 1700. Urban growth, like population growth in the period 1700–1900, was extrodinarily rapid. In 1900, the largest city was London, with 6.5 million inhabitants, and there were 17 cities with over one million residents. Second, the cities that dominate this list were the leading cities from the richest countries. London, New York, Paris, Berlin, Chicago, Vienna, and Tokyo are all found on the list. Further down, we see Manchester, Birmingham, Philadelphia, Boston, Liverpool, Hamburg, and the Ruhr area, all centers of industrialization in the western world.

There were several large agglomerations in relatively poor places in 1900: Beijing, Kolkata, Mumbai, and Rio de Janeiro. But note that none approached the size of the leaders like London or New York. Comparing 1900 to 1700, one can also see that their growth in this period was not on the same scale as the richer cities. Beijing increased only from 700,000 to 1.1 million in the same 200 years that London went from 600,000 to 6.5 million. Istanbul had only 900,000 residents in 1900, up from 700,000 in 1700. Rapid city growth by 1900 was a feature of rich countries, not of poor ones.

This trend continued into 1950, where the top cities remained those in relatively advanced nations. New York, Tokyo, and London were the largest agglomerations, but the remainder of the list contains Chicago, Los Angeles, Detroit, Boston, Birmingham, and Manchester. What we can see, though, is the very beginnings of mega-city growth in relatively poor countries. Kolkata and Shanghai both had more than four million inhabitants in 1950, putting them in the top 10 cities in the world in 1950. Rio de Janeiro, Mexico City, Mumbai, Cairo, and Beijing were all over 2 million inhabitants, reflecting incredible growth for most of them from earlier periods.

By 2015, notice that the nature of the largest cities is dramatically different. First, the absolute scale of cities are 3-5 times larger than in 1950, again representing the combination of rapid population growth and increased urbanization rates. Second, the composition of the list is now dominated by cities in poor countries. Only Tokyo, New York, Los Angeles, Paris, Osaka, and London are in what we would typically term rich countries. Instead we see cities such as Delhi, Sao Paulo, Mumbai, and Mexico City. Countries that rank at the bottom in development levels have cities present on this list, such as Dhaka (Bangladesh), Manila (Philippines), Karachi (Pakistan), Lagos (Nigeria), and Kinshasa (DRC). Each of those poor cities have at least 11 million people, making them larger in absolute size than nearly every city in the world in 1950.

The shift of the largest cities in the world from rich countries to poor countries can be more easily seen in Figure 1. This plots the number of cities with more than one million inhabitants for two groups, developed countries (based on their 2013 GDP per capita) and developing countries. As can be seen, in 1700 there were no cities in either set of countries with one million inhabitants. In 1900 and 1950 nearly all of the million-plus sized cities were in currently developed nations. This switches dramatically between 1950 and 2015, however, and this reversal is projected to increase well into this century. By 2030, the UN expects there to be close to 400 million-resident cities in developing countries, versus only 250 in developed nations.[1] Most of the mega-cities of the world are now, and will continue to be, in relatively poor countries, representing a significant switch in the pattern seen historically.

This change in the composition of the largest cities is only going to be exacerbated in the future. Column (1) of Table 2 shows the 30 largest agglomerations in 2015 ranked by their projected growth rate. As can be seen, this list is led by the primarily by poor mega-cities such as Lagos (Nigeria), Kinshasa (DRC), and Dhaka (Bangladesh). If we turn instead to column (2), this ranks all cities over 5 million inhabitants by their projected growth rate from 2015-2030. Here we see that the fastest growing cities are again some of the poorest; Dar es Salaam (Tanzania), Luanda (Angola), and Abidjan (Ivory Coast) are all expected to grow at over 3 per cent per year, which will take them into the top 30 in absolute city size by 2030.

It is interesting to compare these projected growth rates of poor mega-cities with the growth rates of existing rich mega-cities. Looking at the bottom of column (1) in Table 2, one can find current top-30 mega-cities like London, Paris, New York, and Tokyo. Their growth rates are all under 1%, and Tokyo is expected to actually shrink in size. These cities will be passed by a number of poor mega-cities in absolute size over the next two decades, as their growth has slown down over time and they appear headed for a steady state size. One can see this by

[1] We find a similar pattern if we only consider agglomerations over 5 million people, or the absolute population living in mega-cities (defined either as greater than 1 million or greater than 5 million), or the proportion of population living in mega-cities (again, for either more than 1 million or more than 5 million).

looking at column (2), which ranks all cities by their projected growth rate, and none of the rich wold cities make the list.

This rise of poor mega-cities is reflected in shifts in the relationship between income per capita and urbanization over time. Figure 2 plots the urbanization rate against GDP per capita for three years, 1910, 1950 and 2010, along with linear fits to the relationship. As can be seen, there has been a distinct shift up in the relationship over time. That is, urbanization rates are much higher in 2010 for any given level of income per capita than they were in 1910 or 1950.

In 1910, the poorest countries of the world (i.e. log GDP per capita less than 7, roughly $1,100) all had urbanization rates at or below 10%. By 1950 countries with the same low level of income per capita had urbanization rates of around 20%. But by 2010, though, the urbanization rate for this level of income is around 30%, with several outliers reaching urbanization rates of nearly 50%.

For each year, the positive relationship of income per capita and urbanization remains, and the slopes are statistically indistinguishable from each other. But clearly there has been a shift in the level of this relationship. A different way of interpreting the leftward shift over time in Figure 2 is that countries tend to be *poorer* in 2010 than in 1950 or 1910 for a given urbanization rate. The combination of the information in Figures 1 and 2, as well as Table 1 shows us that mega-cities have grown disproportionately large in the very poorest countries. Urbanization does not imply the same level of living standards that it did in the past.

3. THE DEMOGRAPHICS OF POOR MEGA-CITIES

The rapid population growth of poor mega-cities is different from that experienced during earlier urbanization episodes. The poor mega-cities of today grow in large part through natural increase, along with in-migration. Historically, in-migration tended to be a more important source of new city dwellers as the rates of natural increase were quite low in urban areas, typically because of high mortality rates.

This can be seen clearly if we compare the crude birth rate (CBR) and crude death rate (CDR) of cities across different historical eras. In Figure 3 we plot CBR against CDR for a collection of 14 pre-Industrial Revolution cities, such as ancient Rome, Renaissance Florence, London in the 17th century, and Delhi in the 19th century. As can be seen, the cities all lie on or below the 45-degree line, indicating that birth rates were below death rates, and the cities experienced negative rates of natural increase. Their growth could occur only through in-migration. Those that do lie above the line (Berlin in the 1810s, Beijing in the 19th century, Delhi in the 19th century) are barely above the line, and the crude rate of natural increase is very low.

These data do not give a full account of the effects of major disease outbreaks. Florence in the 15th century (after the plague) has a slightly positive crude rate of natural increase, lying above the 45-degree line. However, in 1348 during the plague years the CDR in Florence was roughly 500, and similar crude death rates of 25-50% of population were seen across European cities (Miskimin, 1975). All the points in Figure 3 thus represent "normal" periods, but each was at times afflicted by similar shocks to mortality, so that the average CDR over long periods of time was likely higher than indicated. Overall, pre-industrial cities tended to experience negative rates of natural increase, and grew only through in-migration.

Figure 4 shows similar data for cities during the Industrial Revolution (with red crosses representing the prior information). Here we can see that while crude birth rates are roughly on the same level with the pre-industrial era, the crude death rate has declined slightly. There remain several cities below the 45-degree line (e.g. London in the 1750s, Amsterdam in the 1850s, Tokyo in the 1900s) but most cities are now experiencing some small positive rate of natural increase. These cities were growing not only through in-migration, but now in part through some increase in their own population. Note, though, that crude death rates remain in the range of roughly 15-40 for these cities.

If we now turn to the post-war period, and the 1960s in particular, we can see in Figure 5 a distinct change in city demographics. Again, the earlier data is plotted with crosses for comparison. Focus first on the collection of relatively rich cities in the lower left, such as London, New York, Paris, and Tokyo. These cities are already very large, as shown in Table 1. But their crude birth rates have fallen along with their crude death rates, and so their rate of natural increase (CBR minus CDR) remains relatively muted, similar in size to that seen during the Industrial Revolution. Overall, it is apparent from Figures 3-5 that historically cities were moving "down" the 45 degree line as they grew, but never straying far from it. Natural increase in these cities was never particularly large, even if the absolute size of CDR and CBR were. As these places developed, the CDR and CBR drop in tandem, keeping the rate of natural increase small.

In comparison are the nascent poor mega-cities of the developing world in the upper left of the figure, well above the 45 degree line, and distinctly different from the historical pattern. In the immediate post-war era these cities differ from earlier eras in one very distinct way: their crude death rates are very low. Lagos, Karachi, and Jakarta all have crude death rates in the 1960s that are roughly similar to those seen in London, New York, or Paris in the same year. The crude birth rates in these cities are large, but not substantially larger than those seen in pre-industrial or industrial revolution era cities. Developing cities in the 1960s were "shifted left" in Figure 5 compared to their historical peers. This led to very large rates of natural increase compared to the prior norms for emerging mega-cities, and this is driven primarily by the substantially lower crude death rates. For example, in the African mega-cities in the figure, crude rates of natural increase are roughly 30, or 3% per year. Absent migration, this implies that these cities double in size roughly every 20 years.

This difference continues into the more contemporary period of the 2000s, which is plotted in Figure 6. Rich mega-cities such as New York, Paris, and Tokyo remain in roughly the same position as in the 1960s, with low crude death and crude birth rates. The poor mega-cities of the developing world have also shifted down to lower crude birth rates, and this makes it more apparent that crude birth rates in these poor mega-cities do not differ from those in earlier eras. However, the crude death rates in these poor mega-cities are much lower than the historical comparisons, for the most part falling below 10 per thousand. Thus in the 2000s poor mega-cities continued to have extremely rapid rates of natural increase. These findings are consistent with those of Jedwab, Christiaensen & Gindelsky (2014), who document the high rates of natural increase for modern developing countries in the urban sector as a whole, as opposed to the specific cities we examine.

The deviation of the developing countries in Figures 5 and 6 from the historical norms of low rates of urban natural increase appears due, in large part, to what Acemoglu & Johnson (2007) refer to as the *international epidemiological transition*. Following 1940, there was a rapid improvement in health (particularly in infant mortality) in many developing countries. This was due to the availability of new treatments (e.g. antibiotics) and vaccines, a world-wide effort by the World Health Orgnization and others to provide access to those new treatments and vaccines, and a focus on rapid dissemination of public health innovations to developing countries (Stolnitz, 1955; Davis, 1956; Preston, 1975). Combined, these changes led to substantial decreases in crude death rates. In turn, health conditions in many developing countries in the post-war era are much better than were conditions in currently developed nations at similar stages of development (Acemoglu & Johnson, 2007).

As the epidemiological transition raised crude rates of natural increase in developing mega-cities, it brought those rates up to the levels seen in rural areas, another anomaly compared to historical experience. In the Industrial Revolution, the crude rate of natural increase in the cities we study was only 3.4 per thousand. For the same time periods, the rural crude rate of natural increase in the respective countries averaged 15.4, the difference coming both through higher birth rates and lower death rates in rural areas (see Web Appendix for sources). By the 1960's however, the crude rate of natural increase in the largest cities was 22.6, on average, with nearly all of that increase coming through a decline in the CDR. During the same period, the crude rate of natural increase in *rural* areas in the same countries was 22.3. So while rates of natural increase were larger in both mega-cities and rural areas, the advance in mega-cities was so substantial that by the 1960s these cities had rates matching those of rural areas.

This can be seen most clearly in Figure 7. Here we plot the urban CRNI for historical and modern cities from the prior figures against the rural CRNI for the associated country in the same time period. A lack of sufficient data on rural areas limits this sample to fewer cities than we have in the prior figures. The observations in the figure are distinguished as developing based on current World Bank classifications.

As can be seen, observations from developed countries lie almost exclusively below the 45 degree line, indicating that urban CRNI was well below rural CRNI. The exceptions to this have very low levels of CRNI in both areas, and are for cities such as New York, Los Angeles, and Paris in the 2000s. The remaining points, which cover London in the 17th century, a number of European cities in the 19th century, and other rich cities in the early part of the 20th century, all lie well below the line. Rural rates of natural increase were much higher than urban rates in these historical cities.

In constrast, the poor mega-cities in developing countries all tend to cluster near the 45 degree line, indicating similar rates of urban and rural CRNI. For many cities the urban CRNI is higher than in the associated rural area. The implication of this is that for these developing cities urbanization did not exert any downward pressure on aggregate population growth, unlike the historical experience where the shift of population into cities acted as a drag on overall population growth. The exceptions to this include mega-cities in China and India in the 2000s. The general pattern, however, is that poor mega-cities tend to have urban rates of natural increase much closer (or higher) than their associated rural areas, while historically cities had much lower rates of natural increase than rural areas.

It is primarily those rapid rates of natural increase that drive the growth of the poor mega-cities, not simply in-migration. Figure 9 plots the projected growth rate from 2015-2030 of the 30 largest cities against their crude rate of natural increase in the 2000s. The relationship is clearly positive, which indicates that crude rate of natural increase is a significant driver of overall city population growth. For the poorest cities in Sub-Saharan Africa and Asia, the crude rates of natural increase are roughly 20-30 per thousand, and this is associated with their expected rapid growth. The Chinese cities (Beijing, Tianjin, Shanghai, Guangzhou, Chongqing) are the main outliers to this relationship, with relatively low rates of natural increase, but relatively high expected growth rates due to in-migration. Cities that are expected to stagnate in size (Moscow, Tokya, Osaka) all have crude rates of natural increase of roughly zero.

These crude rates of natural increase correlate closely with income levels in 2000, as seen in Figure 8. Natural increase is extremely high in the Sub-saharan and Asian cities located in the poorest countries (e.g. Kinshasa, Dar es Salaam, Dhaka, and others). Natural increase falls regularly as countries get richer. We view Figure 8 as representing an equilibrium outcome that our model is capable of explaining. Poor countries have high rates of natural increase, which in turn makes their city size so large that Malthusian forces kick in and keep living standards low, which perpetuates the high rate of natural increase. The opposite holds in the rich mega-cities, where the high standard of living keeps natural increase low, which allows them to maintain the high standard of living.

The last point to make regarding demographics is that the high rates of natural increase do not imply necessarily a large supply of workers, as much of the population will be made up of children with either zero or a low labor supply. Figure 10 plots the dependency ratio (the ratio of those under 14 and 65-plus to the population aged 15-64) against the crude rate of natural increase. Note that the rapidly growing poor mega-cities have very high dependency ratios, on the order of 60% of their working population. While there are several richer cities with relatively high values (Moscow, Tokyo, Osaka) due to large elderly populations, in general the dependency ratio is lower the smaller the crude rate of natural increase. Thus the cities with slower growth maintain a larger fraction of their population as workers, meaning that for their size, they are likely to be more productive than poor mega-cities of the same size.

Overall, it is the distinct demographics of developing countries seen in all the figures that are the source of our explanation for their move into a poor mega-city equilibrium. As we will show below, the rapid rate of natural increase in these cities has two effects. First, the rapid natural increase implies that they very quickly hit the Malthusian congestion effects in urban areas. Second, the high rates of natural increase lowers the possible Malthusian steady state living standard. Rapid natural increase thus pushes wages lower, which in turn implies that natural increase remains high, and the poor mega-citites remain stuck in a low-wage equilibrium where they grow without bound. Hence the demographic shock of very low mortality rates in the post-war era contributes to the arrival and persistence of these poor mega-cities.

4. A MODEL OF MALTHUSIAN URBANIZATION

We pull together several strands of literature to create the model here. First, we consider a model that has simple Malthusian population dynamics, where for very low income levels increases in income result in faster population growth. Second, our model has two separate sectors - urban and rural - as we want to explicitly model the distribution of population in the two areas. This set-up also incorporates the typical non-homotheticicities in demand for rural goods (e.g. food), such that there is a low income elasticity for rural goods, and productivity increases will tend to expand the urban sector. Finally, we adopt explicit microfoundations for urban production that display both agglomeration and congestion, as emphasized in the urban literature. The possibility of congestion in the urban areas will generate the possibility of "Malthusian urbanization" where incomes are driven down as urbanization increases.

In the model we use "city" and "urban sector" inter-changably, implying that the economy has only one city. This is for convenience only, and not necessary for the results. Allowing for multiple cities, with or without a distribution of sizes, would complicate the exposition, but not change the ultimate results.

4.1 Fertility, Mortality, and Equilibrium Income

We begin by specifying the population growth of both sectors. We presume that individuals in either sector have an identical response of population with respect to income, but that there is a level difference between the sectors, denoted u for urban and r for rural,

$$n_{ut} = \phi w_t - d_u \tag{1}$$

$$n_{rt} = \phi w_t - d_r. \tag{2}$$

Here, n_{ut} and n_{rt} are the net additions to population from the two sectors. $\phi > 0$ captures demographic behavior related to income, with the wage in both sectors equal to w_t (we wil be assuming labor mobility between sectors). ϕ captures both the fertility and mortality response to income, and in our setting we have assumed that this leads to an overall net positive relationship of population growth with income.

An alternative would be to allow for a function $\phi = \phi(w_t)$, and allowing non-linearity to the relationship of wages and population growth, ultimately introducing some kind of demographic transition at particularly high levels of w_t with $\phi'(w_t) < 0$. What we require for our model is simply that at very low wages, $\phi'(w_t) > 0$, and that as w_t gets very large $\phi(w_t) > d_u$ and $\phi(w_t) > d_r$ so that population growth remains positive.

The term d_u is a demographic adjustment in urban areas, while d_r is a demographic adjustment in rural ones. One natural interpretation of these terms is that they capture differential mortality rates between sectors, but they could also pick up differential fertility levels between sectors as well. These demographic adjustments are not indexed by time, as we will examine those as static shifters of population growth. In particular, we will think of the epidemiological transition as representing a distinct downward shift in both d_u and d_r.

Total population in period $t+1$, N_{t+1}, will depend on the net population growth rate of the two sectors,

$$N_{t+1} = N_t + N_{ut} n_{ut} + N_{rt} n_{rt} \tag{3}$$

where $N_t = N_{ut} + N_{rt}$ and N_{ut} and N_{rt} are the urban and rural populations at time t, respectively. Letting $u_t = N_{ut}/N_t$ be the urbanization rate, we can write

$$N_{t+1} = N_t + N_t u_t n_{ut} + N_t(1-u_t) n_{rt} \tag{4}$$

$$= N_t \left(1 + \phi w_t - u_t d_u - (1-u_t) d_r\right). \tag{5}$$

For a given urbanization rate, u_t, we can solve for the income level that results in a steady state population, $N_{t+1} = N_t$,

$$\overline{w} = \frac{u_t d_u + (1-u_t) d_r}{\phi}. \tag{6}$$

While this demographic setting is relatively simple, it is worth establishing several key comparative static results regarding the level of $\overline{w}$ and its relationship to mortality rates and urbanization. Given the definition of the Malthusian steady state wage, $\overline{w}$, in equation (6), the following relationships hold with respect to urbanization, u_t,

$$\partial\overline{w}/\partial u_t \begin{cases} >0 & \text{if } d_u > d_r \\ <0 & \text{if } d_u < d_r \\ =0 & \text{if } d_u = d_r \end{cases} \tag{7}$$

and the following relationships hold with respect to the adjustment terms,

$$\begin{aligned} \partial\overline{w}/\partial d_u &> 0 \\ \partial\overline{w}/\partial d_r &> 0. \end{aligned} \tag{8}$$

What they show is that the effect of urbanization on the Malthusian steady state wage depends on the relative demographic adjustment rates. If $d_u > d_r$, then higher urbanization implies lower population growth and hence a higher Malthusian steady state wage. This is the typical effect seen in most Malthusian models, and one could think of it as capturing higher mortality in urban areas. In cases where $d_u < d_r$, however, increased urbanization can lower living standards because it raise population growth rates. The second set of statics just establishes that if either d_u or d_r rise, the steady state wage will rise.

The value of $\overline{w}$ defines the income at which the economy is at a steady state population size. Whether the economy ever actually reaches this steady state depends on how wages respond to the size of population, which depends on assumptions regarding production in both the rural and urban sectors.

4.2 Rural Production and Urbanization Rates

In a typical Malthusian model, there is some fixed resource (e.g. agricultural land) in the rural sector that implies income per worker declines with population size, and combined with the above population process, this results in a Malthusian steady state.

Here, to highlight the Malthusian influence of urban areas, we relax completely the assumption of a fixed resource in the rural sector. This stark assumption is made for clarity only, and the model can be adapted to include a fixed resource in the rural sector without changing the underlying logic.

From the supply side, rural production is

$$Y_{rt} = A_r N_{rt} \tag{9}$$

so that rural labor has a constant return. From the demand side, we assume that all N_t individuals demand precisely $\bar{c}_r$ of the agricultural good at all times. Hence there is a very stark non-homotheticity in demand for rural goods, and an income elasticity of zero. Setting supply equal to demand yields

$$A_r N_{rt} = \bar{c}_r N_t \tag{10}$$

which can be combined with the identity $N_t = N_{ut} + N_{rt}$ to solve for the urbanization rate

$$u_t = 1 - \frac{\bar{c}_r}{A_r}. \tag{11}$$

This simple setup implies that the share of workers in the urban area depends on rural productivity, A_r. Note that A_r only influences the urbanization *rate*, u_t. The absolute size of the urban population (and the overall population) will depend on the fertility and mortality relationships given in the prior section, and how wages respond to the absolute size of the urban and/or total population.

Putting the urbanization rate into the expression for $\overline{w}$ from above yields

$$\overline{w} = \frac{d_u + \frac{\bar{c}_r}{A_r}(d_r - d_u)}{\phi}. \tag{12}$$

Similar to what was seen in the prior section, the effect of rural productivity on $\overline{w}$ depends on the relative size of the demographic adjustment in the two areas. When $d_u > d_r$, then increases in A_r generate a higher level of $\overline{w}$ by pushing people into low-population growth urban areas. In contrast, when $d_u < d_r$, higher rural productivity lowers $\overline{w}$ by pushing people in what is now a high population growth urban area.

4.3 Production in Urban Areas

At this point, we know the value of $\overline{w}$, the wage that will result in constant population size, but we have not established whether in fact the economy will ever reach this steady state value. In the typical Malthusian model w_t would fall as N_t increased due to a resource constraint, and this would ensure a stable steady state. Here we have assumed that there is no resource constraint, so there is nothing in the rural sector to drive down w_t as N_t falls.

In its place we outline a model of urban agglomeration and congestion that will ultimately provide a Malthusian force in the economy. The basic concept is that while increasing urban population, N_u, provides some agglomeration effects, eventually congestion sets in, and further urban population actually will lower output. There is a resource constraint, so to speak, but it occurs because the urban sector becomes too crowded.

We allow for two types of urban production: a formal sector and an informal sector. The distinction is made so that we can capture some of the features of developing mega-cities, which typically include large slums that are not really part of the network of formal urban production, but do feature in the costs of congestion. Both the formal and informal urban sectors are modelled identically, differing only in a set of parameters governing agglomeration and congestion effects. As such, we first describe our model of urban production for any given sector, without sector-specific notation.

4.3.1 Urban Agglomeration

Urban final goods in either sector are produced using a series of intermediate inputs,

$$Y_u = \left(\sum_{i=0}^{M} x_i^{\frac{1}{1+\rho}} \right)^{1+\rho} \tag{13}$$

where x_i is the amount of intermediate good i used and M is the number of intermediate goods used in equilibrium. The elasticity of subsitution between intermediate goods is $(1+\rho)/\rho$, with $\rho > 0$. Letting p_i represent the price of intermediate good i, the inverse demand function for good i is

$$p_i = x_i^{-\frac{\rho}{1+\rho}} Y_r. \tag{14}$$

Each intermediate good is produced by a monopolistically competitive firm using the production function

$$x_i = BL_i - F \tag{15}$$

where B is the productivity of the firm (assumed to be identical across all firms), L_i is the labor used by firm i, and F is a fixed cost for a firm to operate. The fixed costs imply that there are increasing returns to scale in the production of each intermediate good. These increasing returns will ultimately capture the agglomeration effects at work in urban areas, and these will be offset in the aggregate by congestion effects.

The intermediate good firms maximize their profits,

$$\pi_i = p_i x_i - w_u L_i, \tag{16}$$

taking the wage w_u as given, and knowing the inverse demand curve for their good given in (14). This leads to the typical markup over marginal cost, with

$$p_i = (1+\rho)\frac{w_u}{B}. \tag{17}$$

We further assume that intermediate goods firms can enter and exit freely in the urban area, so that profits for any individual intermediate goods firm are driven to zero. Using the production function for firms in (15) and the price given in (17) the only possible level of output consistent with zero profits is

$$x_i = \frac{F}{\rho}. \tag{18}$$

Given this level of output, each firm hires

$$L_i = \frac{1+\rho}{\rho}\frac{F}{B} \tag{19}$$

workers.

As each intermediate good provider is identical, their total demand for labor must equal the total supply of labor in the urban area, L_u

$$\sum_{i=0}^{M} L_i = L_u, \tag{20}$$

which can be solved for the equilibrium number of firms,

$$M = \frac{L_u}{L_i} = \frac{\rho}{1+\rho}\frac{B}{F}L_u. \tag{21}$$

Finally, using (18) and (21) in the production function (13) yields

$$Y_u = A_u L_u^{1+\rho} \tag{22}$$

where

$$A_u = \frac{\rho^{\rho}}{(1+\rho)^{1+\rho}}\frac{B^{1+\rho}}{F^{\rho}} \tag{23}$$

is the aggregate productivity term for the urban sector. Note that output in the urban sector has increasing returns to scale, as $\rho > 0$. Each intermediate good firm operates with a number of workers proportional to the fixed cost. If there are more workers in the urban area, then this allows more firms to operate. More intermediate goods firms increases productivity in the urban final goods sector by allowing them to access a wider variety of inputs. This captures the agglomeration effects at work in urban areas in our model - a larger urban workforce allows greater specialization and therefore higher productivity.

4.3.2 Urban Congestion

To model the congestion associated with higher urban populations, we adopt a simple urban structure. All production takes place at a central point in the city, a central business district, so to speak. Residents of the city live along a line extending both directions from the central business district. There is a time cost to communting to the central business district, equal to 2τ times the distance from the center. As each worker needs to go back and forth each day, the total time cost for a worker at distance j from the city center is $4\tau j$, leaving them with only $1-4\tau j$ units of time left to provide to the labor market.

The distance that each worker has to travel is a function of the number of workers, N_u. Typically, each worker would use up one unit along the line, so that the maximum distance a worker was from the center was $N_u/2$, as workers can live in either direction. Integrating over all the workers we can find the total labor supply

$$L_u = 2\int_0^{N_u/2} (1-4\tau j)\,dj = N_u\left[1-\tau N_u\right]. \tag{24}$$

Here we can see the impact of congestion. Labor supply, L_u is increasing in the number of urban workers, N_u, but only up to a point. Eventually increased urban population becomes so burdensome that the actual labor supplied by workers falls.

To this point we have not said anything about rents. Those workers closer to the city center will supply more labor, and hence earn more. We presume that there is a competitive rental market that in equilibrium ensures that each worker in the city earns, on net, an identical amount, with higher gross earnings offset exactly by higher rents for those living close to the center of the city. The total rents collected are presumed to be spread equally across all urban residents. The purpose of these assumptions is simply to make earnings identical for all workers within a sector, eliminating the need to keep track of earnings distributions within sectors.

We also abstract from the question of competition for land between the rural and urban sectors. That is, we presume that urban area can be expanded costlessly without lowering the land available for rural production. While obviously not realistic, including an explicit market in which land is exchanged for use in the two sectors would complicate the model without adding anything substansive.

4.3.3 Urban Wage Equilibrium

We can now discuss the two urban sectors: formal and informal. The informal sector is the easiest to explain. We take the stark position that $\rho = 0$ and $\tau = 0$. Therefore, there are *no* agglomeration effects in informal production, but also *no* congestion effects. The urban informal sector thus reduces to

$$Y_u^{inf} = A_u^{inf} L_u^{inf} \tag{25}$$

and the income per worker using the informal technology is

$$w_u^{inf} = A_u^{inf}. \tag{26}$$

To be clear, informal *production* does not suffer from congestion effects - one can think of informal workers operating home-based businesses that do not require transportation around the city. Those workers in the informal sector will still cause congestion for the formal sector, however.

The formal sector is presumed to have values of $\rho > 0$ and $\tau > 0$, meaning that it enjoys some agglomeration economies but also is disrupted by congestion effects. That is, workers in the formal sector all have to agglomerate in a central business district, so their labor supply is lowered by having to travel through the city.

In the formal sector, combine the expression for output from (22) with the labor supply equation from (24) to find

$$Y_u^{for} = A_u^{for} N_u^{1+\rho} \left[1 - \tau N_u\right]^{1+\rho} \tag{27}$$

and income per worker using the formal technology is

$$w_u^{for} = A_u^{for} N_u^{\rho} \left[1 - \tau N_u\right]^{1+\rho}. \tag{28}$$

What can be seen here is that the number of urban workers, N_u, influences earnings in the formal sector, and that these earnings form an inverted U-shape. That is, for low levels of N_u earnings are increasing in urban workers as the agglomeration effects outweigh the congestion effect. Eventually, though, when N_u is large enough the congestion effects dominate and more urban workers lower earnings.

The implication of the inverted U-shape in the formal urban sector is that for either very small cities or very large cities, the *informal* sector will offer higher earnings. This can be seen most easily in figure 11, which plots both w_u^{for} and w_u^{inf} against N_u.

It will be useful to establish the following lemma for future use.

Lemma 1. *Given the informal wage rate w_u^{inf} and the formal wage rate w_u^{for} from (28), then*

- *there exists a level $\underline{N}_u$, such that for $N_{ut} < \underline{N}_u$ all workers earn w_u^{inf}*
- *there exists a level $\overline{N}_u$, such that for $N_{ut} > \overline{N}_u$ all workers earn w_u^{inf}*

Proof. The existence of $\underline{N}_u$ and $\overline{N}_u$ can be seen from the fact that $w_u^{for} = 0$ when $N_u = 0$ and $w_u^{for} = 0$ when $N_u = 1/\tau$, and from the fact that it is a continuous function in between those values of N_u. □

If $N_u > \overline{N}_u$, then there are two possibilities for the distribution of workers within urban areas. All workers could be in the informal sector. Alternatively, $\overline{N}_u$ workers can be in the formal sector, forcing earnings down to the informal level w_u^{inf}, while the remaining $N_u - \overline{N}_u$ workers will be in the informal sector. Any continued population growth will lead to an expansion of the informal sector, while the formal sector will remain at the fixed size $\overline{N}_u$.

The equilibrium urban wage at any given time t is denoted by

$$w_{ut} = max(w_u^{inf}, w_{ut}^{for}) \tag{29}$$

which is implicitly a function of N_{ut}, as seen in figure 11.

A last note regarding the equilibrium urban wage and the number of cities. The model is written as if there is a single city in which all urban residents live. That is not a necessary assumption, although it makes the analysis simpler. If there are a fixed number of cities M, then given identical technologies each city will have N_u/M residents, and the wage will be equalized across cities. One could consider a distribution of city sizes by allowing for differences in A_u and/or ρ across cities.

One may also consider the possibility that new cities are founded to avoid the congestion effects present in existing ones. With a judicious choice of the number of cities, one could ensure that each one operates at the maximum possible wage. We do not feel this is a particularly relevant possibility. There is a coordination issue with starting new cities. No individual has any incentive to set out by oneself to start a new city, as they will not enjoy the agglomeration advantages of a city, and the wage will be essentially zero in their new city. It would require a coordinated decision by a sufficiently large group of workers to make starting a new city worth the effort. We are assuming, therefore, that this type of coordination effort is not possible. Considering that we are likely talking about tens of thousands, if not hundreds of thousands or even millions of people having to move at once, this does not appear to be a particularly strong assumption. If we combine that with any kind of preference for staying in one's native city, or with fixed costs to starting new cities, then it is even less plausible that new cities would form in response to congestion effects. For those reasons, we go forward with the assumption that there is a single urban area in the model.

4.4 The Poor Mega-City Equilibrium

We have an economy that has a possible Malthusian mechanism at work in the urban sector. Whether the economy actually comes to rest in a Malthusian steady state with constant population size depends on whether wages ever reach the level $\overline{w}$. To determine the earnings of individuals in the economy, we allow for free mobility of labor between the rural and urban sectors, resulting in a common wage between the two, w_t. If rural goods have a relative price of p_r, we have that

$$w_t = p_r A_r = w_{ut} \tag{30}$$

and the value of p_r will adjust to ensure that this holds. This implies that we can look solely at the urban wage to determine the wage rate.

Given the value of $\overline{w}$ from (12), the urban wage rate in (29), and the assumption that population growth is increasing in w_t ($\phi > 0$), then there are two distinct regimes the economy may find itself in. To see this regimes, first note that we can write the urban wage in (29) as

$$max\left(\frac{w_u^{inf}}{A_{ut}}, N_u^{\rho}\left[1-\tau N_u\right]^{1+\rho}\right) = \frac{w_{ut}}{A_{ut}}. \tag{31}$$

The right-hand side is simply the productivity adjusted urban wage.

The first regime leads to poor mega-cities. It holds when $\overline{w} < w_u^{inf}$, and the informal wage is below the steady state value. In this equilibrium, there is no stable steady state size of the urban population, and therefore no stable size of the overall population. Figure 12 shows this, plotting w_{ut}/A_{ut} against the size of N_{ut}. Here, because the productivity-adjusted wage can never reach $\overline{w}/A_{ut}$ due to the presence of the informal sector, population growth never ceases and the city grows without bound, regardless of the initial level of urban popluation. There never is a Malthusian steady state. We can characterize this equilibrium more formally.

Proposition 1. ***Poor Mega-City Equilibrium:***
If $\overline{w} < w_u^{inf}$ for all t, then the following hold:

(A) $N_{u,t+1} > N_{ut}$ for all t

(B) There exists some time $\overline{t}$ such that for $t > \overline{t}$, $w_t = w_u^{inf}$

Proof. With $\overline{w} < w_u^{inf}$, then given the population process it must be that $N_{u,t+1} > N_{ut}$, so (A) holds. Given (A) and Lemma 1, then eventually $N_{ut} > \overline{N}_u$, and $w_t = w_u^{inf}$. ☐

In words, the propostion says that if $\overline{w}$ is sufficiently small - which depends on the demographic terms d_u and d_r as well as the urbanization rate, u_t - then urban population never ceases to grow. Further, at some point in time, the urban sector will grow so large that only the informal economy becomes viable, as congestion effects in the formal sector wipe out the agglomeration effects.

It is important to note that the poor mega-city equilibrium can occur regardless of the level of A_u. It is irrelevant what the absolute level of urban formal sector productivity is, as the congestion effects in urban areas still limit wages in that sector. Developing countries may have access to high-productivity formal urban technologies, but if their urban natural rate of increase is very large and their urbanization rate is sufficiently large, then they will enter the poor mega-city regime.

This leads to a further corollary regarding the urban formal sector productivity level.

Corollary 1. *If $\overline{w} < w_u^{inf}$ for all t, then*

(A) The maximum possible number of workers in the formal urban sector is $N_u^{for} = 1/\tau$

(B) For all $t > \overline{t}$, urban population N_{ut} is invariant with respect to A_u

(C) For all $t > \overline{t}$, wages are invariant with respect to A_u, as $w_t = w_u^{inf}$

Proof. (A) follows from the nature of the formal sector wage in (28). Regardless of the level of A_u, the formal sector wage will go to zero if there are $1/\tau$ formal sector workers. (B) follows from Proposition 1, as once $N_{ut} > \overline{N}_u$, the formal technology is irrelevant for wages. (C) follows from Proposition 1, which says that eventually the economy will have $w_t = w_u^{inf}$, which is invariant with respect to A_u. □

This corollary shows that if economies find themselves with poor mega-cities, as in Proposition 1, they will eventually have no incentive to increase the size of their urban formal sector, and their earnings will be stagnant even if urban formal sector technology improves. Because of congestion effects, eventually the wage in the formal sector is pinned down at w_u^{inf}, the wage of the marginal worker, regardless of the size of A_u. A Proposition 1 notes, these poor mega-cities continue to grow in absolute size, but this is unrelated to urban productivity, A_u. In this equilibrium, it is thus possible that an economy will have mega-cities that use primarily informal technology even though they have full access to formal sector technology. These cities may have some formal sector contained within them, as noted in the discussion of figure 11. However, as their population grows the formal sector will remain at a maximum size of $\overline{N}_u$ even as N_{ut} increases without bound.

4.5 The Historical Equilibrium

As noted above, the poor mega-city equilibrium is not the only possible outcome. The second regime that we consider conforms to the historical norm, and holds when $\overline{w} > w_u^{inf}$. This will lead to a stable steady state level of urban population (and hence of overall population) and a constant wage equal to $\overline{w}$. In constrast to the poor mega-cities, in the historical equilibrium there is not continuous growth in urban population, but the size of population is directly related to the level of productivity, A_u.

Figure 13 shows the equilibrium in this situation. There are two steady states. There is an unstable equilibrium at N_u^*, and if the economy starts below this level of urban population wages will fall along with the urban population until N_u reaches zero. Unlike a typical Malthusian model based on a rural resource constraint, the urban Malthusian model features the possiblity that the population can get too small to sustain a population. For $N_u < N_u^*$, workers are so poor that population growth is negative, and this *lowers* wages because it removes the benefits of agglomeration in the urban area. While each worker still consumes the subsistence rural good, $\overline{c}_r$, the population will continue to fall to zero over time as population growth was assumed to depend on wages in terms of the urban goods. One could easily modify the model to ensure that population size had some floor it could not go beneath, but what remains true in our setting is that the size of the *urban* population will tend to zero if population is too small.

The second steady state is at N_u^{**}, and this is stable, with population growth acting to force the economy to this point if $N_u > N^*$ in the first place (a topic we return to below). High wages in the urban sector produce rapid population growth over some range, but by increasing the size of the urban sector, this produces congestion effects that create a Malthusian force for falling wages. Eventually, congestion will get so bad that wages are driven down to $\overline{w}$, and the economy is in a steady state, with stagnant population and stagnant wages even though there is no fixed resource in the rural sector.

We make a more formal statement of the equilibrium here.

Proposition 2. ***The Historical City Equilibrium:***
If $\overline{w} > w_u^{inf}$ for all t, then the following hold:

(A) N_{ut} has a steady state of either N_u^{**} or zero

(B) w_t has a steady state of $\overline{w}$

Proof. (A) and (B) both follow from the assumption that $\phi > 0$, and so population growth is rising with respect to w_t, combined with the nature of urban wages in (29). ☐

Whether the economy reaches the stable steady state N_u^{**} depends on whether the initial urban size, N_{u0} is greater than the unstable steady state value of N_u^*. We do not explicitly model the initial value of urban population is greater than N_u^*. For our purposes in comparing existing historical rich mega-cities to today's poor mega-cities, it is sufficient to note that there were rich mega-cities that did meet this initial requirement.[2]

Regardless, we can consider the reaction of the stable steady state to changes in technology. Formally,

Corollary 2. *If $\overline{w} > w_u^{inf}$ for all t, then*

(A) $\partial N_u^{**}/\partial A_u > 0$

(B) The maximum city size is $N_u^{**} = 1/\tau$

(C) All urban workers are employed in the urban formal sector

Proof. (A) can be found from the implicit function theorem, setting the formal sector wage in (28) equal to $\overline{w}$, which is constant. (B) follows given (A), as A_u becomes arbitrarily large, N_u^{**} increases towards $1/\tau$, which is the maximum city size possible while holding $\overline{w} > w_u^{inf}$. (C) is implied by the fact that the steady state wage is $\overline{w} > w_u^{inf}$. ☐

The proposition shows that in historical cities, increasing urban productivity will raise equilibrium city size. Higher A_u allows more people to exist in the city despite the congestion effects. Overall growth in equilibrium city size, N_u^{**}, thus depends on growth of A_u.

This raises an interesting contrast with the poor mega-cities. In the poor mega-cities, urban population grows continuously due to natural increase, but this growth is unaffected by A_u (Corollary 1). For historical cities, urban population growth only occurs due to productivity improvements, but not through natural increase.

[2]If one were interested in eliminating the possibility of collapsing to zero, then one possibility would be to modify d_u to be a function of N_u itself, perhaps indicating that urban mortality rates rise with urban size. As N_u went to zero and d_u fell, this could make $\overline{w} < w_u^{inf}$, so that there was no unstable steady state.

5. THE ORIGIN OF POOR MEGA-CITIES

Our model shows that Malthusian forces can operate in the urban sector of the economy, and that this opens up the possibility for poor mega-cities. In particular, the poor mega-city equilibrium holds when $\overline{w} < w_u^{inf}$, and the city's wages never drop to a value that stabilitizes population growth. The key to poor mega-cities lies in the interaction of their specific demographic patterns with the Malthusian nature of urban congestion. Developing countries after World War II had two distinct features. First, as documented in section 3 the absolute level of the crude death rate was very low relative to historical experience, particularly in urban areas but also in rural ones. In terms of our model, this indicates that d_u and d_r were very low. Second, urban mortality fell so much that there ceased to be a large gap between urban and rural rates of natural increase, implying that $d_u = d_r$.

The absolute drop in d_u and d_r means that the Malthusian steady state wage, $\overline{w}$, fell for any given level of urbanization, u_t, as in equation (6). Second, the equality of d_u and d_r means that $\overline{w}$ was invariant with respect to the urbanization rate, as shown in (7).

The drop in $\overline{w}$ creates the possibility of entering the poor mega-city equilibrium, as described in Proposition 1. The additional fact that $\overline{w}$ is invariant to urbanization means that there is no force capable of pushing $\overline{w}$ above the informal wage, and allowing these countries out of the poor mega-city equilibrium.

Increasing urbanization rates - which recall are driven by rural productivity A_r - therefore do not necessarily imply any gains in income per worker in poor mega-cities. Further, Corollary 1 implies that poor mega-city wages are unaffected by changes in formal sector technology, A_u. Productivity growth in either sector ultimately has no effect on living standards in the poor mega-city equilibrium. This conforms to the data presented in Figure 2 that shows even very poor countries in 2010 with relatively large urbanization rates. The only possible source of income growth for these cities is if the informal technology wage, w_u^{inf}, rises. It does not seem unreasonable to believe that most research and development efforts are *not* engaged in this sector, and that the growth of the informal urban wage is much slower than the pace of productivity growth in the formal urban sector or even the rural sector.

A last point regarding the poor mega-cities is that as time continues, it becomes harder and harder for them to possibly transition out of the poor equilibrium. Even if the level of $\overline{w}$ were to somehow rise above w_u^{inf}, this does not instantly send the poor mega-city to the new steady state wage of $\overline{w}$ and steady state size of N_u^{**}. Wages would remain stagnant at w_u^{inf} for the amount of time it took N_{ut} to shrink back to $\overline{N}_u$ (where the formal technology becomes viable again). The longer poor mega-cities grow, the longer they will have to remain at the informal wage.

In sum, poor mega-cities arise because the shock to their rates of urban natural increase put too much pressure on the urban formal technology. This creates rapid growth in the absolute number of urban residents, which creates a Malthusian-like congestion effect in the urban formal sector. This drives the wages of that sector down to the informal level. At that point, continued population growth is translated into additional informal sector workers, and everyone earns the minimum w_u^{inf}. The mega-city continues to grow in size, but not in living standards, and its size is unrelated to formal sector productivity.

As we noted in the introduction, the existence of these poor mega-cities stands in contrast to the experience of many developed nations of today, who saw very large urban agglomerations come into being while also experiencing rising living standards. The key difference is in the underlying demographics. Prior to the epidemiological transition, historical cities had two characteristics leading them towards the more favorable equilibrium. First, absolute mortality rates were relatively high, meaning d_u and d_r were large, and as indicated in (6), implying that $\overline{w}$ was very large. Second, cities were "killers" compared to rural areas, meaning that $d_u > d_r$.

The combination of these two facts allowed cities to enter the historical equilibrium. With $\overline{w}$ kept high by the high mortality rates, we have the Malthusian implication that steady state wages were relatively high as well. High enough, in most cases, that $\overline{w} > w_u^{inf}$ and the cities could reach a stable steady state size of N^{**}.

Further, though, the fact that $d_u > d_r$ meant that the urbanization process - driven by rural productivity growth, A_r - was a distinct *positive* for wages. Recall from (7) that $\overline{w}$ rises with u_t when $d_u > d_r$. When urban mortality is high, then shifting people into the urban sector lowers the population growth rate, and this in turn raises living standards in a Malthusian setting such as we have. So even if some of these cities perhaps began with $\overline{w} < w_u^{inf}$, and were headed for the poor mega-city equilibrium, the process of urbanization itself would have helped lift them out of that equilibrium.

With historical killer cities, we have that increases in rural productivity were positive for living standards, unlike the poor mega-cities of today. Similarly, as documented in Corollary 2, increases in A_u raised the equilibrium size of these historical cities. Historically, productivity generated larger cities, higher urbanization rates, and higher incomes. This contrasts with the situation in the poor mega-cities of today, where productivity increases in the rural sector increase the urbanization *rate*, productivity increases in either sector have *no* effect on incomes, and absolute city size grows solely because of natural increase.

6. EXTENSIONS AND ADDITIONS

In developing our model of poor mega-cities, we made assumptions to highlight the most important forces. There are several extensions that we discuss here that would allow for more nuance in the model.

6.1 Agglomeration and Congestion in the Informal Sector

We assumed that $\tau = 0$ and $\phi = 0$ for the informal sector, making the informal wage constant with respect to N_{ut}. This implies that poor mega-cities can grow without bound. An alternative is to allow the informal sector to have agglomeration and congestion effects, only more muted than in the formal sector. Let $\tau^{inf} < \tau^{for}$ and $\rho^{inf} < \rho^{for}$. The urban wage is then

$$w_{ut} = max\left(A_u^{inf}(N_{ut})^{\rho^{inf}}\left[1-\tau^{inf}N_{ut}\right]^{1+\rho^{inf}}, A_u^{for}(N_{ut})^{\rho^{for}}\left[1-\tau^{for}N_{ut}\right]^{1+\rho^{for}}\right). \quad (32)$$

The type of equilibrium an economy reaches is still dependent on the level of $\overline{w}$. But now both types - historical and poor mega-cities - will reach a point where $\overline{w} = w_{ut}$.

One can establish that for sufficiently high values of d_u and/or d_r, the equilibrium will occur using the formal technology, as in the original model. For sufficiently low values of d_u and d_r, the poor mega-city equilibrium will occur using the informal model. The major difference from before is that poor mega-cities would also conceivably reach a steady state size. The poor mega-city steady state size would be definitively *larger* than the historical steady state, given the assumptions regarding τ. With a low value of τ^{inf}, the city can expand to a greater size before congestion drives wages down to $\overline{w}$ in the poor mega-city.

Any growth in formal sector technology, A_u^{for}, would still have no effect on poor mega-city size or on wages. However, advances in A_r may influence both size and income in poor mega-cities by altering the urbanization rate, and with it the size of $\overline{w}$.

6.2 Rural Resources

To this point we've assumed that the rural sector is free of any resource constraint. If there is a resource constraint, then this complicates the solution, but the implications remain.

If we do have a more traditional fixed resource in this sector, then production would have decreasing returns to labor, as in

$$Y_{rt} = A_r N_{rt}^{\beta}, \quad (33)$$

where we've set the stock of resources to one, without loss of generality. In this case we can still consider the Malthusian steady state equilibriums. Assuming again that demand for rural output is fixed at $\overline{c}_r$ for each worker, we have

$$N_t \overline{c}_r = A_r N_{rt}^{\beta} \quad (34)$$

setting demand equal to supply. Noting that $N_t = N_{ut} + N_{rt}$ and $u_t = N_{ut}/N_t$, we can write the urbanization rate as

$$u_t = 1 - \left(\frac{N_t^{1-\beta}\overline{c}_r}{A_r}\right)^{1/\beta}, \quad (35)$$

which gives urbanization as a function of total population.

In the urban sector, adopt the assumption from the prior subsection regarding agglomeration and congestion in the informal sector, shown in equation (32). In a Malthusian steady state, it must be that $w_{ut} = \overline{w}$, and this implies that

$$w_{ut} = \frac{u_t d_u + (1-u_t)d_r}{\phi} \tag{36}$$

given the definition of $\overline{w}$ from (6). Note that given the description of urban wages in w_{ut}, and the fact that $N_{ut} = u_t N_t$, this expression also relates the urbanization rate to the overall size of the population. Equations (35) and (36) can be solved together for the steady state equilibrium values of overall population and the urbanization rate.

Similar to our baseline model, whether the economy has an equilibrium using the formal sector, with relatively high wages and small city sizes, or the informal sector, with relatively low wages and large city sizes, depends on the demographic terms d_u and d_r. Higher values for those indicate slower population growth, and hence make an equilibrium using the formal technology more likely. Low values for d_u and d_r, such as occurred after the epidemiological transition, make using the informal technology more likely, resulting in a poor mega-city.

6.3 The Take-off to Sustained Growth

In the model, historical cities reached an equilibrium with a wage of $\overline{w}$, and this wage is increasing with urbanization, given that cities were killers in the sense of having $d_u > d_r$. So as agricultural productivity, A_r, increased, incomes in the historical cities would increase as well. However, this effect can only last up to a point, as eventually urbanization reaches a limit of $u_t = 1$.

Sustained growth beyond that requires a similar structure to other models of unified growth. Namely, we have to allow for some kind of demographic response to increasing wages that allows population growth to slow down. If historical cities could reach, through agricultural productivity and urbanization, some cut-off level of w_t such that their demographic response to further increases was negative (i.e. $\phi'(w_t) < 0$), then sustained growth would be a possibility.

Consider a cutoff level of w_t above which the population growth rate goes to zero. If wages reach this cut-off, and city size ceases to grow, then growth in the urban wage is simply $\Delta w_{u,t+1}/w_{ut} = \Delta A_{u,t+1}/A_{ut}$. So long as there is growth in A_{ut}, then there is growth in wages. Specifying the exact nature of the growth in that urban productivity term is beyond the scope of the paper, but note that the formal urban sector involves individual profit-making firms, so one could build a model of research and development that is based on those profits.

One note on how this compares to poor mega-cities. If we imagine that some exogenous force reduces population growth in the poor mega-cities to zero despite their low wage, note that they will *not* experience any kind of sustained growth. Stuck as they are with too many people to make the formal sector viable, improvements in the formal sector technology have no effect on wages in poor mega-cities *even if their populations cease growing*.

6.4 Urban Consumption Amenities

One could distinguish urban production agglomeration and congestion effects from urban consumption agglomeration and congestion effects. These consumption agglomerations are the amenities associated with larger cities - different types of restaurants, more opportunities for entertainment, pure pleasure with diversity - and the consumption congestion effects would arise through crowding or transportation effects. If there are urban consumption amenities, then this creates a separate reason for people to want to live in urban areas, aside from the purely income-based motive we have in the model.

If we did allow there to be some consumption amenities, then the equilibrium movement of labor between sectors would be based on equating *utility* between sectors, as opposed to *incomes*. This would not changes the possibility of poor mega-cities, but might suggest an additional factor leading countries into that equilibrium. If people had sufficiently strong preferences for urban consumption amentities, this would raise urbanization rates for any given population size, and lower urban wages. For sufficiently strong preferences, this could tip a country into the poor mega-city equilibrium.

6.5 Investing in Congestion Technology

Historical cities in our model are limited in absolute size at $N_u = 1\tau$, as the formal technology is not viable with higher population size. Poor mega-cities are not constrained in size, but do not operate the formal technology. Of course, many of the historical cities - London, Tokyo, New York - have grown to be nearly as large as poor mega-cities today, and have done so without declining wages.

In terms of the model, we think of these modern rich cities as having invested in reducing τ; imagine subways, sewage systems, and electrical grids. By lowering τ, the maximum size of a cities formal sector expands. This would allow for larger size without having a negative impact on wages.

Given that these congestion-reducing investments are typically scale-dependent (there is no use in having only one subway station), we could think of there being some kind of revenue threshold that a city must meet before it can afford these types of investments. If we let

$\overline{R}$ be that threshold revenue level, then for the rich historical cities, they reach this cut-off if $\overline{w}N_u^{**} \geq \overline{R}$. Improvements in either agricultural productivity, which raise $\overline{w}$ through urbanization, or urban productivity, which raise N^{**}, would allow the city to reach the cut-off to investing in lower τ.

For poor mega-cities, the threshold is met if $w_u^{inf} N_{ut} \geq \overline{R}$. So after sufficient population growth, they should be able to make these investments, assuming that informal wages can be mobilized or taxed in the same manner as formal sector wages. However, note that even if poor mega-cities reach this threshold, and *can* invest in reducing τ, it isn't necessarily clear that they *will* invest in reducing τ. Lowering τ expands the maximum size of the formal sector, and hence increases the possible number of formal sector workers. But if urban population is higher than the maximum possible number of formal workers, the wage will remain at w_u^{inf}, and there is no incentive to invest in alleviating congestion. Further, even if they do lower τ, it will only temporarily raise wages as population growth will still be positive given $\overline{w} < w_u^{inf}$, meaning that eventually the city will grow back to the point that wages are at the informal level.

7. CONCLUSION

The appearance of poor mega-cities in developing countries is a departure from historical experience. Prior to the post-war 20th century, the largest urban agglomerations tended to be in the richest countries. Now we are witnessing both very large absolute city sizes in poor developing countries, but also relatively high urbanization rates for their level of income per capita.

To explain the departure of the poor mega-cities from the typical pattern we develop a model of Malthusian urbanization. Urban areas are subject to negative congestion effects from increased population in addition to any positive agglomeration effects that may arise. This congestion creates a Malthusian setting where population and living standards are negatively related. Combined with endogenous population growth, cities have a tendency to drive down living standards to a steady state level that is consistent with a stable population size. In the model, there are two type of equilibria possible, and the selection into the equilibria depends on the natural rate of increase in the urban population. For high rates of urban natural increase, such as those seen following the epidemiological transition after World War II, cities grow so large that the congestion effects overwhelm the agglomeration effects. In constrast, in the high-mortality cities of the past, cities were constrained from growing too large and they reached a Malthusian equilibrium with relatively high living standards.

The arrival of poor mega-cities is a consequence of mortality interventions that raised urban rates of natural increase. This poor equilibrium has several pessimistic implications. Poor mega-cities will tend to grow without bound with access to an informal production technology. This unlimited growth exacerbates the congestion issues in using modern, formal urban technologies and so these cities will tend to use only the informal, low productivity technology even if they have access to the formal one. Further, the continued growth in poor mega-cities means that mitigating congestion issues so that the high productivity formal technologies are viable is increasingly expensive and unlikely to occur.

Our work shows that developing countries are not free of Malthusian constraints even if they have abundant natural resources or access to international markets. Even without limits on resources like agricultural land, the absolute size of mega-cities in these countries creates a Malthusian setting which acts as a brake on their potential growth.

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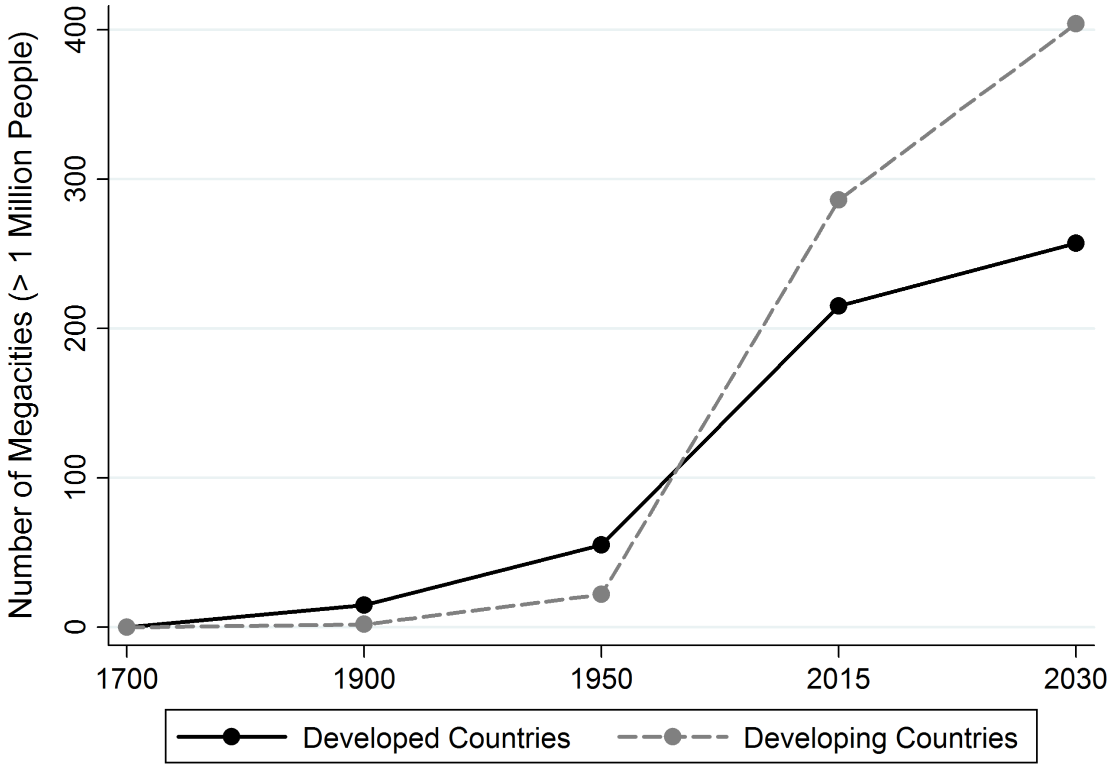
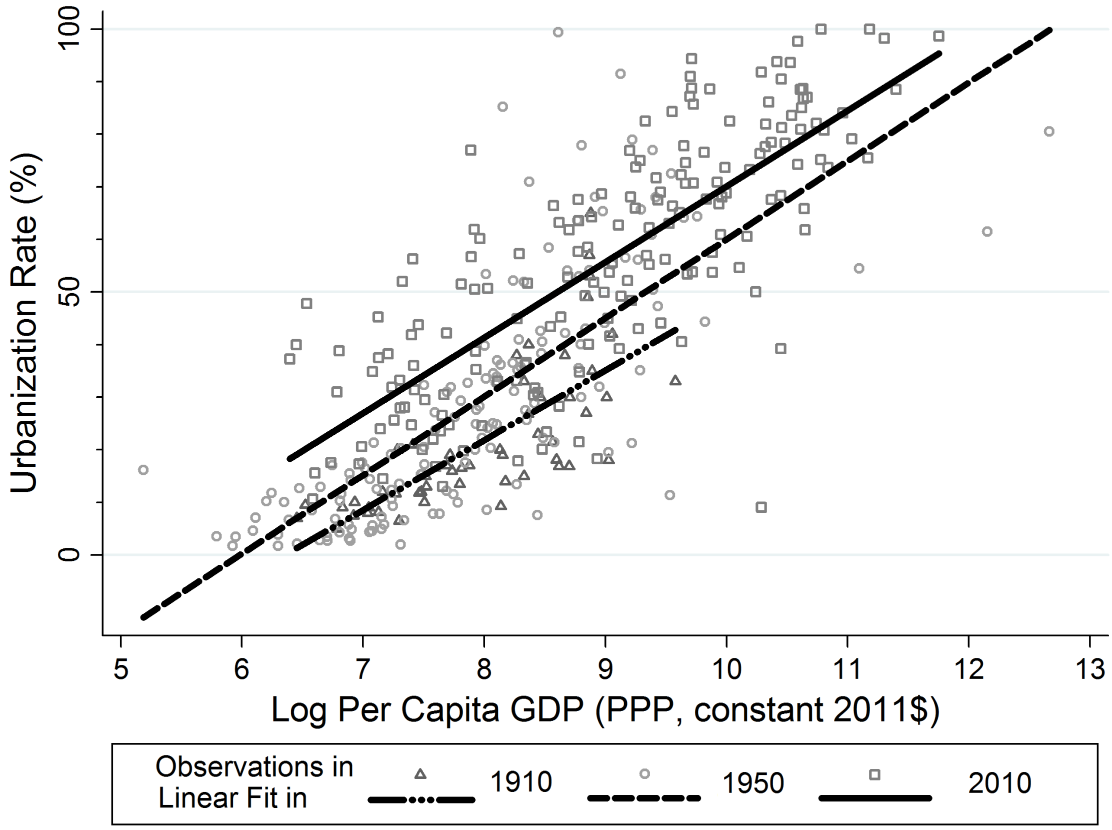
FIGURE 1: NUMBER OF MEGACITIES (ABOVE 1 MILLION PEOPLE) IN DEVELOPED AND DEVELOPING COUNTRIES, 1700-2030



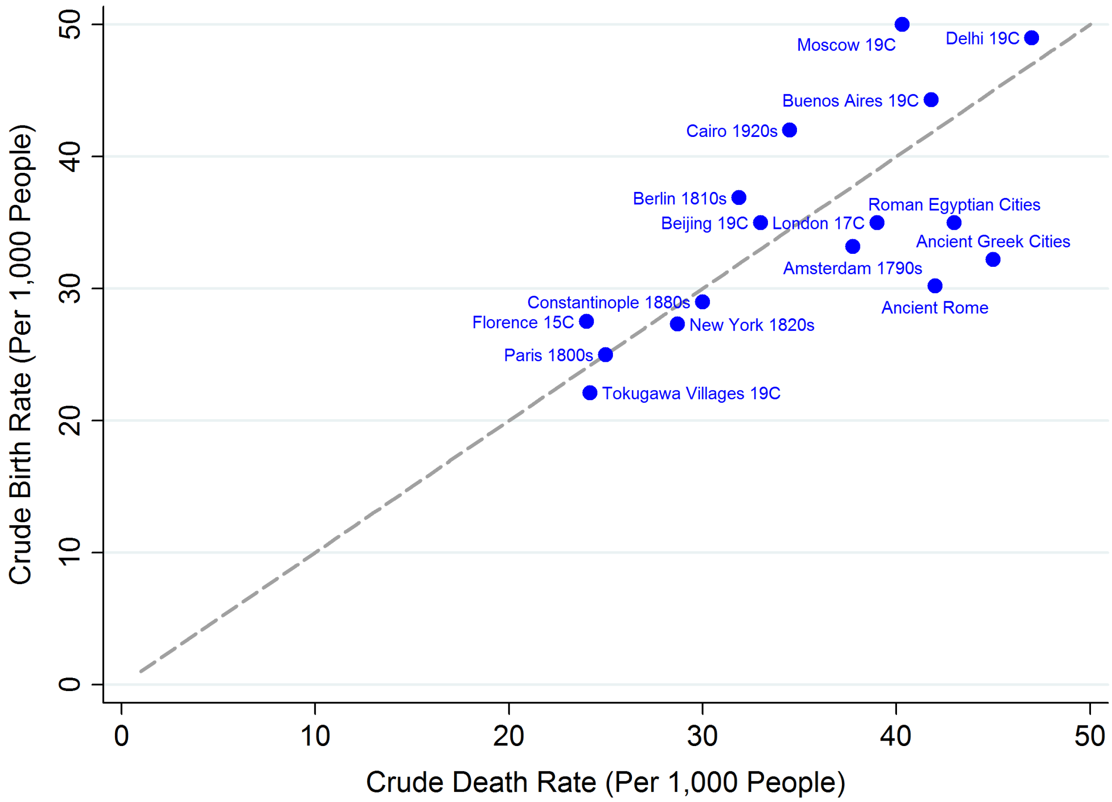
Notes: This figure shows the number of urban agglomerations of at least 1 million inhabitants in developed countries and developing countries in 1700, 1900, 1950, 2015 and 2030. Developed (developing) countries are countries whose GDP per capita (PPP, constant 2011 international $) is above (below) $12,476 in 2013. Sources: Maddison (2008), World Bank (2013) and United Nations (2014).

FIGURE 2: INCOME AND THE URBANIZATION RATE ACROSS TIME



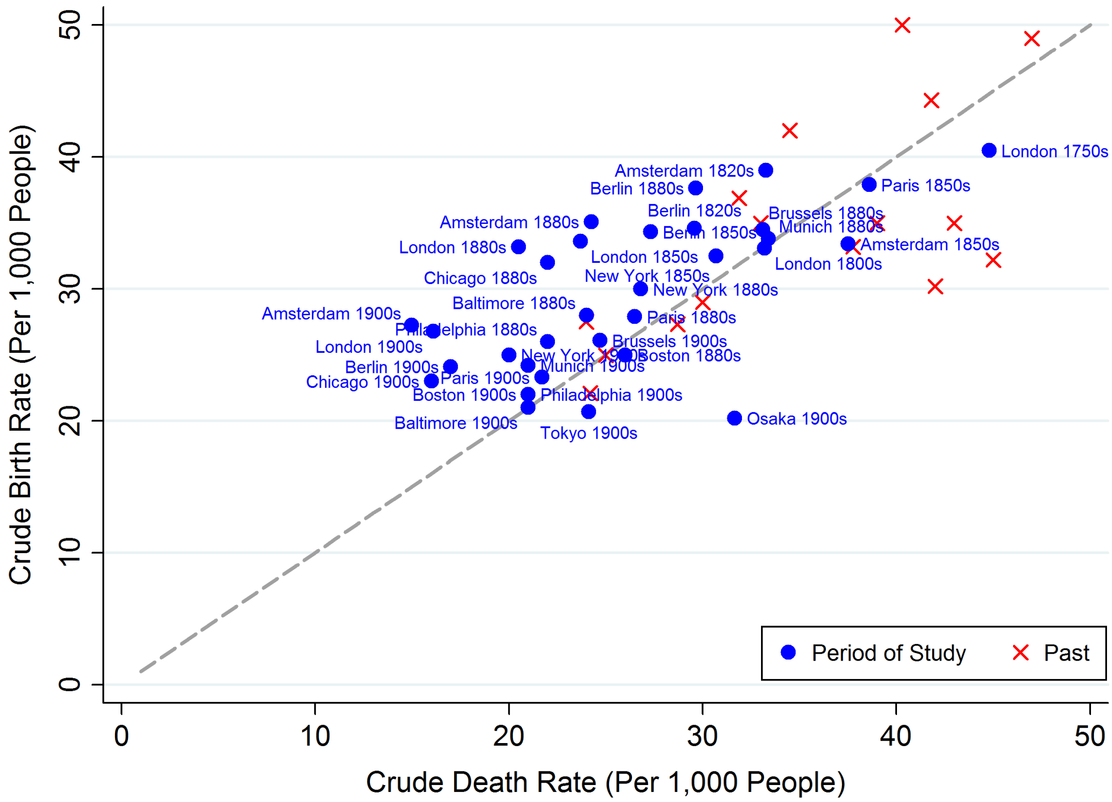
Notes: This figure plots the linear relationship between the urbanization rate (%) and log per capita GDP (PPP, constant 2011$) for 52 countries in 1910, 142 countries in 1950 and 177 countries in 2010. We have fewer observations for the two earlier years, because of missing historical data on urbanization and/or per capita GDP. Sources: Bairoch (1988), Malanima & Volckart (2007), Maddison (2008), World Bank (2013) and United Nations (2014).

FIGURE 3: CITY CRUDE BIRTH AND DEATH RATES, PRE-INDUSTRIAL ERA



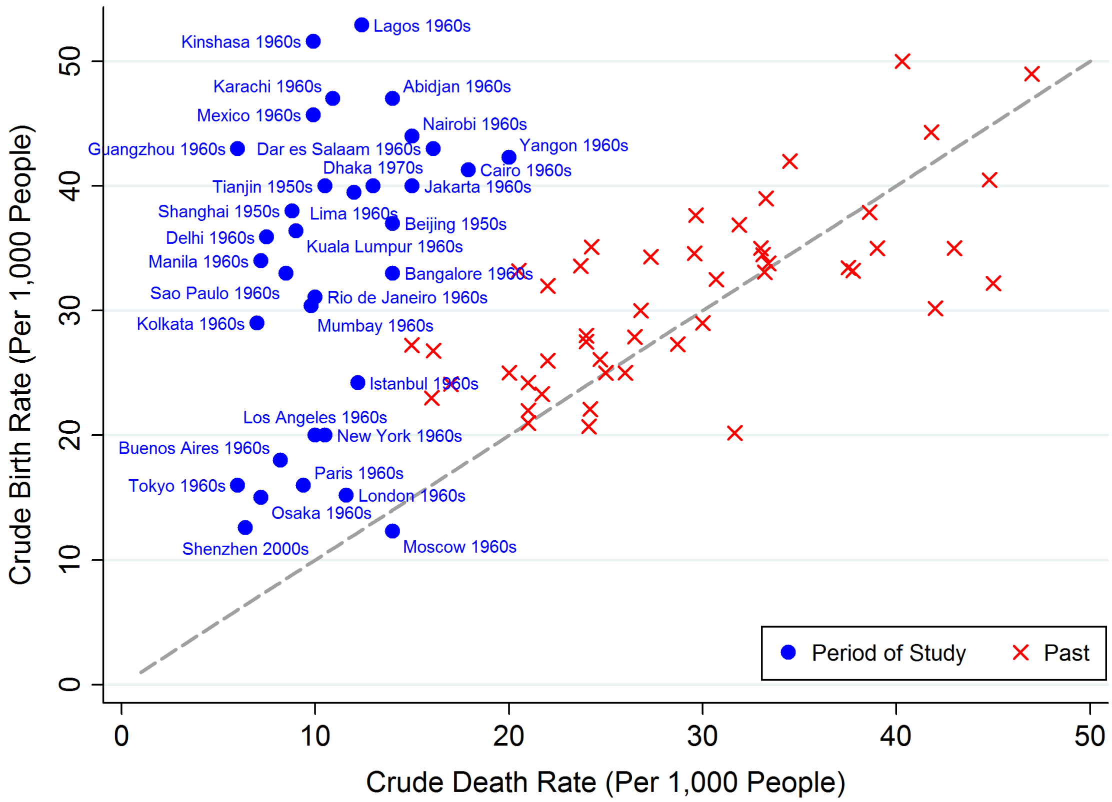
Notes: This figure shows the crude birth rate (per 1,000 people) and the crude death rate (per 1,000 people) for 14 pre-Industrial Revolution cities of Ancient Greece, the Roman Empire, the Italian Renaissance, the Qing Dynasty in China, the Tokugawa Shogunate in Japan, the Moghul Empire in India, the Ottoman Empire in Turkey, and Europe and North America before the Industrial Revolution. See Web Appendix for data sources.

FIGURE 4: CITY CRUDE BIRTH AND DEATH RATES, INDUSTRIAL REVOLUTION



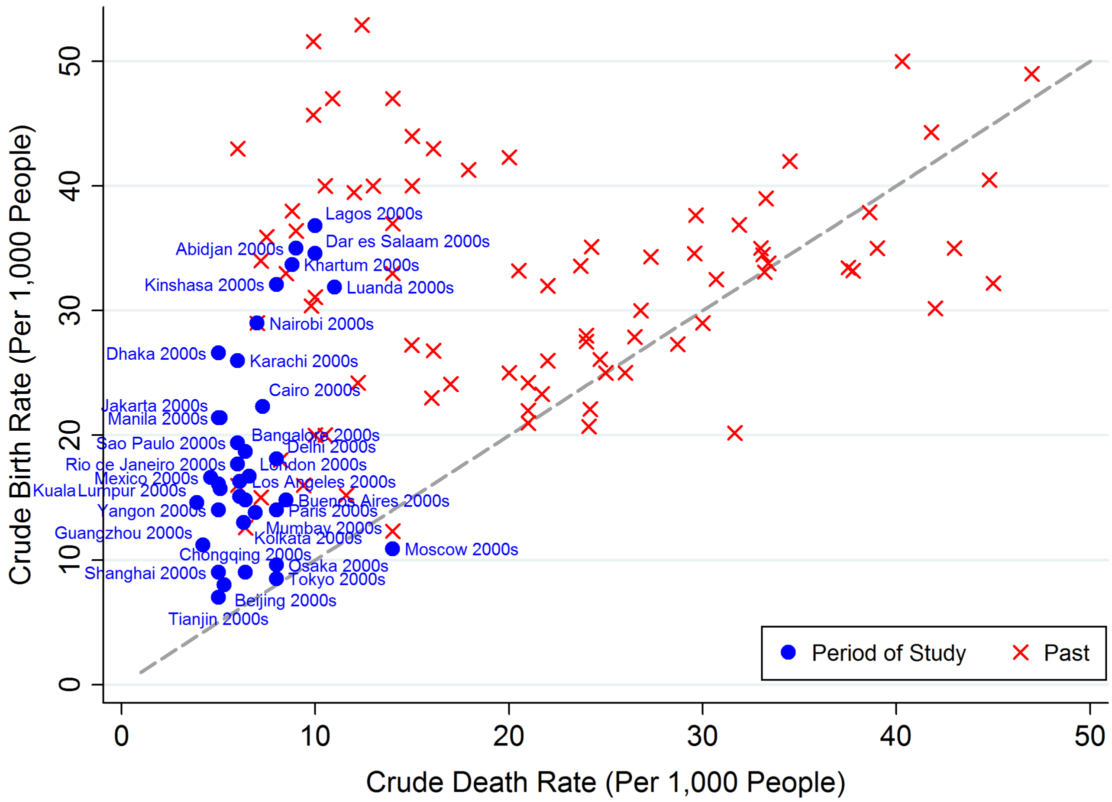
Notes: This figure shows the crude birth rate (per 1,000 people) and the crude death rate (per 1,000 people) for 33 city-decade observations. The 13 European and North American cities are Amsterdam, Baltimore, Berlin, Boston, Brussels, Chicago, London, Munich, New York, Osaka, Paris, Philadelphia and Tokyo. We used the observations for the following decades, when the data was available: 1750s, 1800s, 1820s, 1850s ,1880s and 1900s. The observations denoted by a large X show the megacities of the pre-industrial era from figure 3. See Web Appendix for data sources.

FIGURE 5: CITY CRUDE BIRTH AND DEATH RATES, 1960s



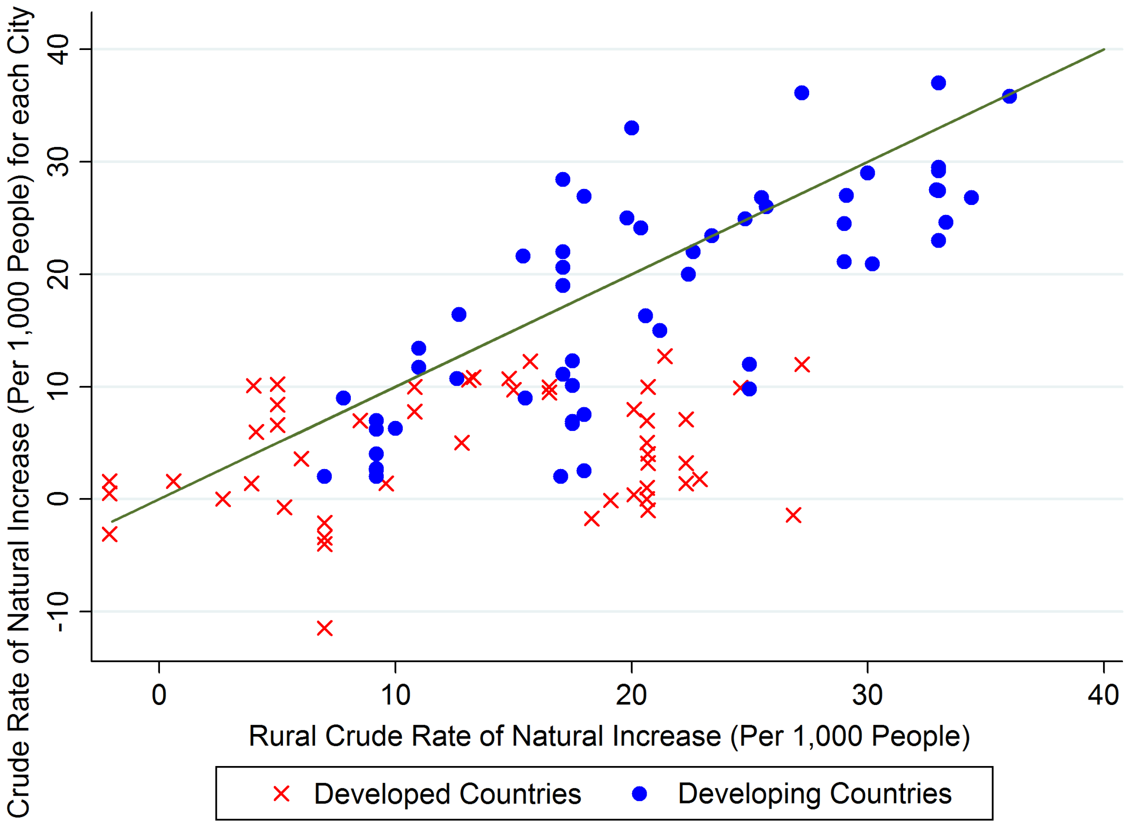
Notes: This figure shows the crude birth rate (per 1,000 people) and the crude death rate (per 1,000 people) for 26 megacities in the 1950s-1970s, for the following continents: Africa (N = 6), America (5), Asia (12) and Europe (3). The observations denoted by a large X show the megacities in the pre-industrial and industrial revolution era from figures 3 and 4. See Web Appendix for data sources.

FIGURE 6: CITY CRUDE BIRTH AND DEATH RATES, 2000s



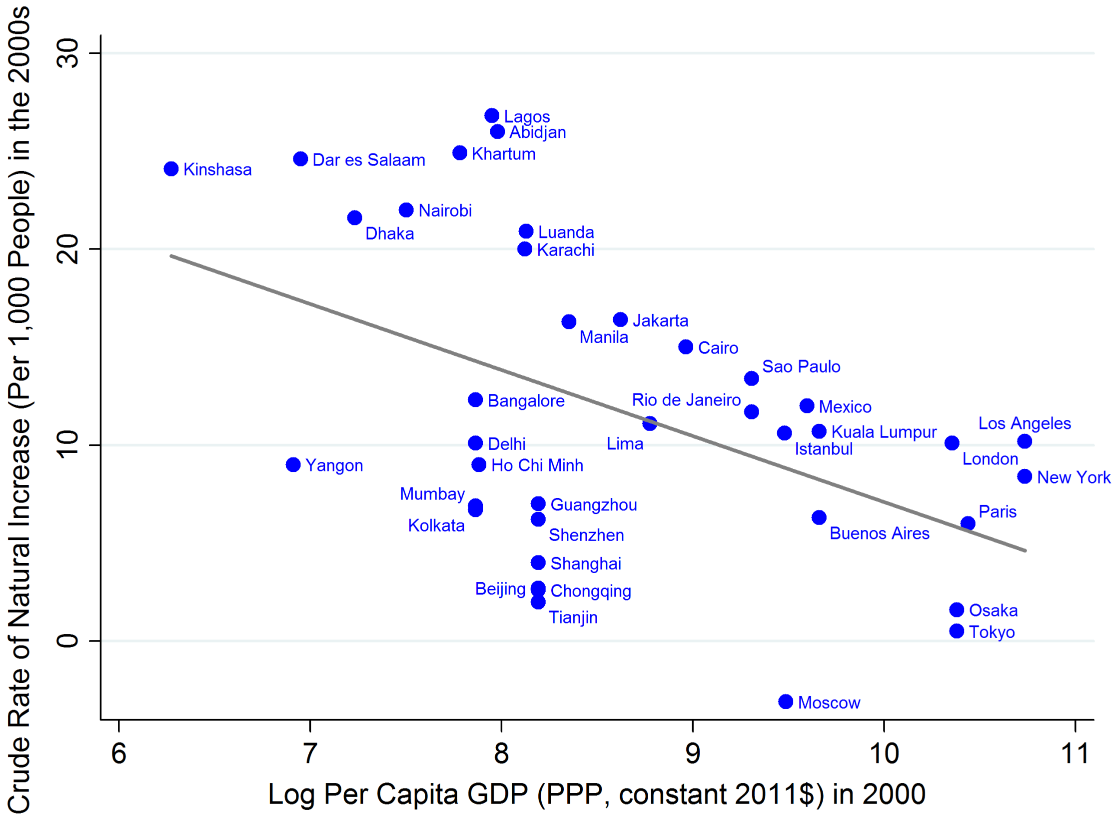
Notes: This figure shows the crude birth rate (per 1,000 people) and the crude death rate (per 1,000 people) for 27 megacities in the 2000s, for the following continents: Africa (N = 7), America (5), Asia (12) and Europe (3). The observations denoted by a large X show megacities of the pre-industrial, industrial revolution, and 1960s, from figures 3,4, and 5. See Web Appendix for data sources.

FIGURE 7: CRUDE RATES OF NATURAL INCREASE IN CITIES AND RURAL AREAS



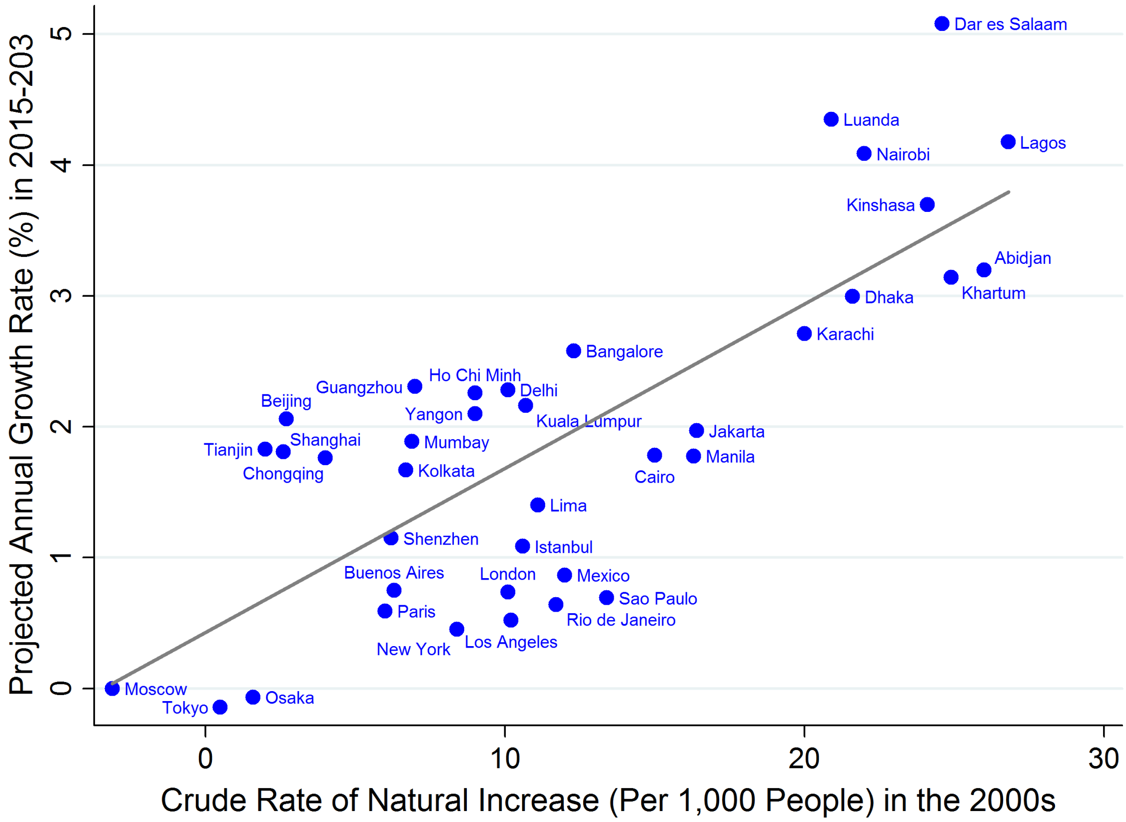
Notes: This figure shows the crude rate of natural increase in cities and the associated crude rate of natural increase in the rural sector of the respective country. This is done for 68 cities from the modern era - including the 1960s and 2000s - and the pre-modern era - meaning observations from prior to the early 20th century. The cities are all from the same list of megacities used in prior figures. See Web Appendix for data sources.

FIGURE 8: INCOME LEVEL AND CITY CRUDE RATES OF NATURAL INCREASE, 2000s



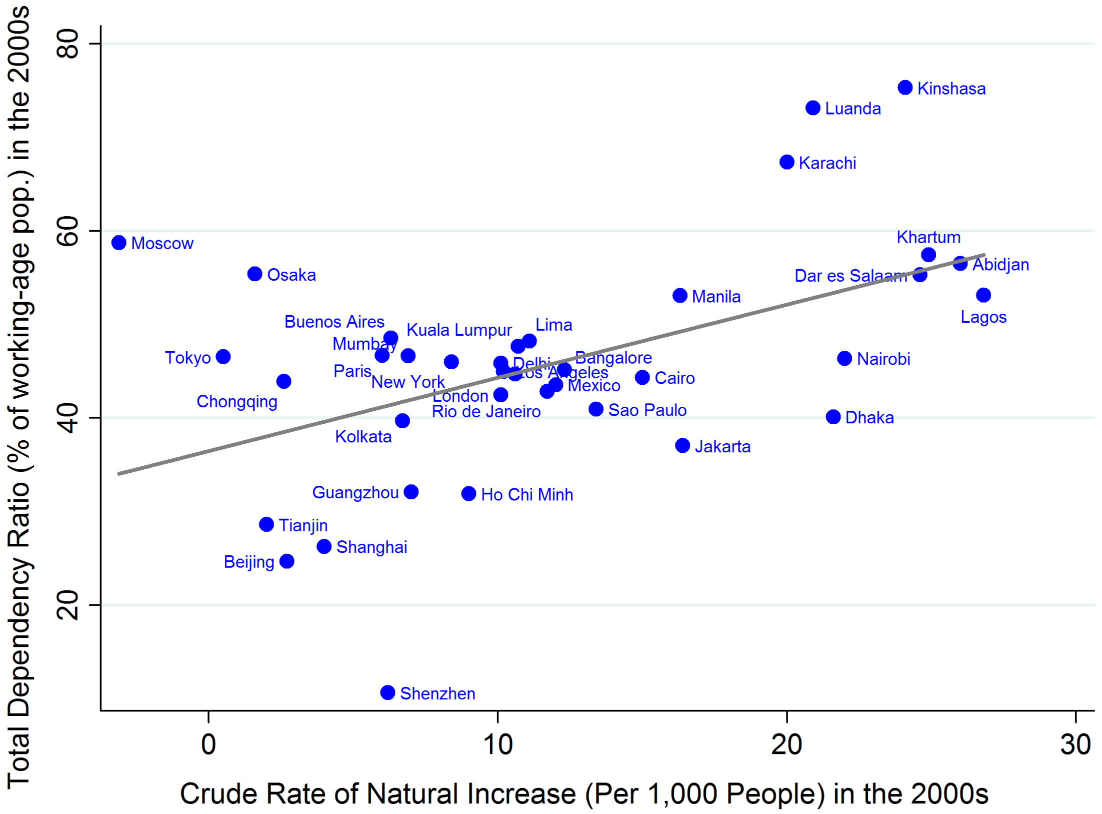
Notes: This graph shows the relationship between the crude rate of natural increase (per 1,000 people) for 38 megacities in the 2000s and log per capita GDP (PPP, constant 2011 $) for the country of the megacity in 2000. See Web Appendix for data sources.

FIGURE 9: CRUDE RATES OF NATURAL INCREASE AND PROJECTED CITY GROWTH, 2015-2030



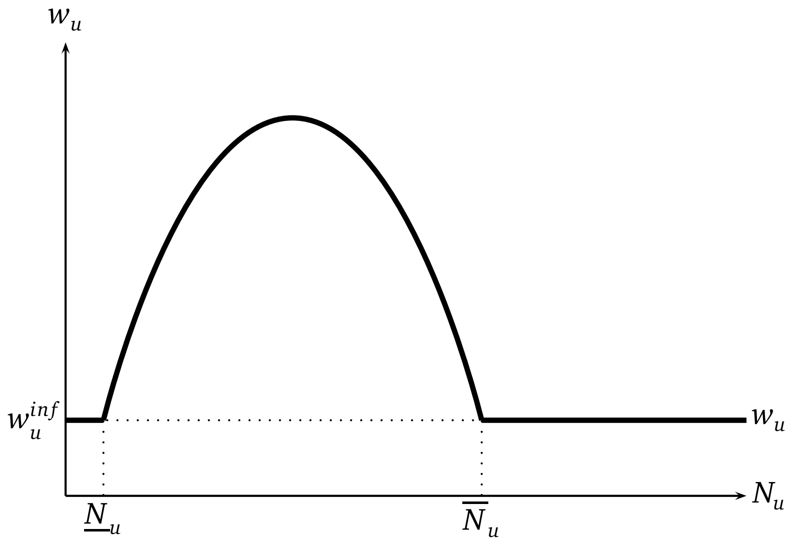
Notes: This graph shows the relationship between the projected annual growth rate (%) in 2015-2030 and the crude rate of natural increase (per 1,000 people) in the 2000s, for 38 megacities of Table 2. Data is missing for the other megacities of the table. We use **?** to obtain the projected growth rate of each city. These rates are based on non-linear extrapolation given the rates of growth pre-2015, and are thus estimated independently of the rates of natural increase. See Web Appendix for data sources.

FIGURE 10: CRUDE RATES OF NATURAL INCREASE AND DEPENDENCY RATIO, 2000s



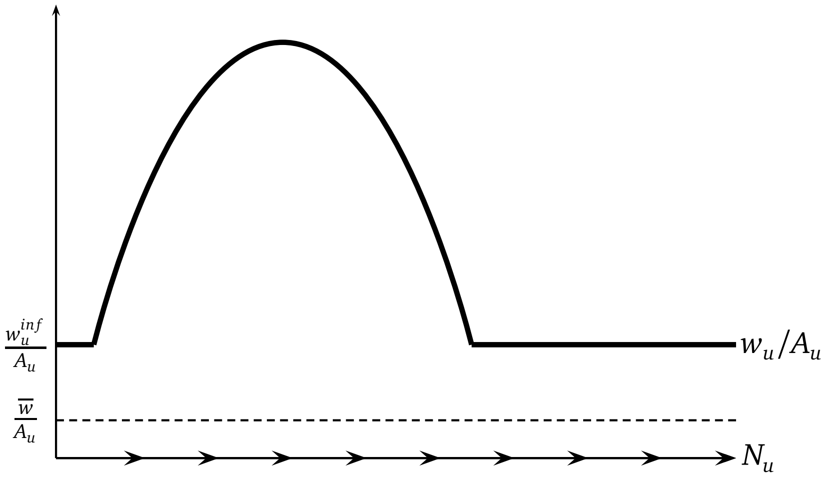
Notes: This graph shows the correlation between the dependency ratio (%) – the ratio of the population of 0-14 y.o. plus above-65 y.o over the population of 15-64 y.o. – and the crude rate of natural increase (per 1,000 people) for 37 megacities of Table 2 in the 2000s. See Web Appendix for data sources.

FIGURE 11: URBAN WAGE WITH INFORMAL SECTOR



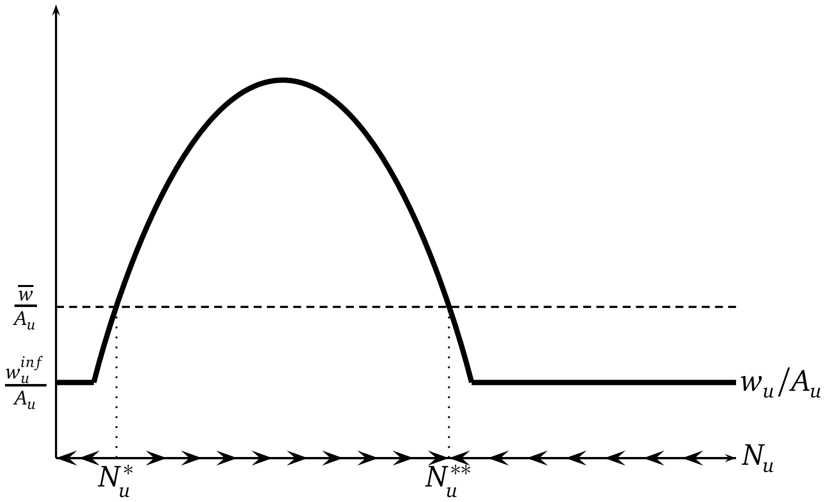
Notes: This figure shows the urban wage when there are two possible sectors of production. The formal sector displays both agglomeration ($\rho > 0$) and congestion ($\tau > 0$) in production, which creates the inverted U-shape with respect to urban population, N_u. The informal sector has neither agglomeration nor congestion ($\rho = 0$ and $\tau = 0$), and so is invariant to N_u. The values $\underline{N}_u$ and $\overline{N}_u$ represent the cut-offs between which workers will choose to work in the formal sector.

FIGURE 12: **EQUILIBRIUM CITY SIZE WHEN** $\overline{w} < w_u^{inf}$



Notes: This figure shows the equilibrium size of urban population when the Malthusian equilibrium wage ($\overline{w}$) is smaller than the informal wage (w_u^{inf}). With $w_u^{inf} > \overline{w}$, urban population size is always growing, and hence there is no steady state value of N_u, and the urban sector grows without bound.

FIGURE 13: **EQUILIBRIUM CITY SIZE WHEN** $\overline{w} > w_u^{inf}$



Notes: This figure shows the equilibrium size of urban population when the Malthusian equilibrium wage ($\overline{w}$) is larger than the informal wage (w_u^{inf}). With $w_u > \overline{w}$, urban population size is growing, and with $w_u < \overline{w}$, urban population is falling. Given this, there is an unstable steady state at N_u^*, and a stable steady state at N_u^{**}.

TABLE 1: WORLD'S LARGEST MEGACITIES (MILLIONS), 1700-2015

Rank	1700		1900		1950		2015	
1	Istanbul	0.7	London	6.5	New York	12.3	Tokyo	38.0
2	Tokyo	0.7	New York	4.2	Tokyo	11.3	Delhi	25.7
3	Beijing	0.7	Paris	3.3	London	8.4	Shanghai	23.7
4	London	0.6	Berlin	2.7	Paris	6.3	Sao Paulo	21.1
5	Paris	0.5	Chicago	1.7	Moscow	5.4	Mumbay	21.0
6	Ahmedabad	0.4	Vienna	1.7	Buenos Aires	5.1	Mexico	21.0
7	Osaka	0.4	Tokyo	1.5	Chicago	5.0	Beijing	20.4
8	Isfahan	0.4	St. Petersburg	1.4	Kolkata	4.5	Osaka	20.2
9	Kyoto	0.4	Manchester	1.4	Shanghai	4.3	Cairo	18.8
10	Hangzhou	0.3	Philadelphia	1.4	Osaka	4.1	New York	18.6
11	Amsterdam	0.2	Birmingham	1.2	Los Angeles	4.0	Dhaka	17.6
12	Naples	0.2	Moscow	1.1	Berlin	3.3	Karachi	16.6
13	Guangzhou	0.2	Beijing	1.1	Philadelphia	3.1	Buenos Aires	15.2
14	Aurangabad	0.2	Kolkata	1.1	Rio de Janeiro	3.0	Kolkata	14.9
15	Lisbon	0.2	Boston	1.1	St. Petersburg	2.9	Istanbul	14.2
16	Cairo	0.2	Glasgow	1.0	Mexico	2.9	Chongqing	13.3
17	Xian	0.2	Osaka	1.0	Mumbai	2.9	Lagos	13.1
18	Seoul	0.2	Liverpool	0.9	Detroit	2.8	Manila	12.9
19	Dacca	0.2	Istanbul	0.9	Boston	2.6	Rio de Janeiro	12.9
20	Ayutthaya	0.2	Hamburg	0.9	Cairo	2.5	Guangzhou	12.5
21	Venice	0.1	Buenos Aires	0.8	Tianjin	2.5	Los Angeles	12.3
22	Suzhou	0.1	Budapest	0.8	Manchester	2.4	Moscow	12.2
23	Nanking	0.1	Mumbay	0.8	Sao Paulo	2.3	Kinshasa	11.6
24	Rome	0.1	Ruhr	0.8	Birmingham	2.2	Tianjin	11.2
25	Smyrna	0.1	Rio de Janeiro	0.7	Shenyang	2.1	Paris	10.8
26	Srinagar	0.1	Warsaw	0.7	Roma	1.9	Shenzhen	10.7
27	Palermo	0.1	Tientsin	0.7	Milano	1.9	Jakarta	10.3
28	Moscow	0.1	Shanghai	0.6	San Francisco	1.9	London	10.3
29	Milan	0.1	Newcastle	0.6	Barcelona	1.8	Bangalore	10.1
30	Madrid	0.1	St. Louis	0.6	Glasgow	1.8	Lima	9.9

Sources: Chandler (1987) and United Nations (2014).

TABLE 2: PROJECTED ANNUAL GROWTH RATES FOR MEGACITIES (%), 2015-2030

Rank	(1) Among Top 30 Cities in 2015		(2) Among All Cities ≥ 5 Millions in 2015	
1	Lagos	4.2	Dar es Salaam	5.1
2	Kinshasa	3.7	Luanda	4.4
3	Dhaka	3.0	Lagos	4.2
4	Karachi	2.7	Kinshasa	3.7
5	Bangalore	2.6	Abidjan	3.2
6	Guangzhou	2.3	Khartoum	3.1
7	Delhi	2.3	Dhaka	3.0
8	Beijing	2.1	Surat	2.9
9	Jakarta	2.0	Karachi	2.7
10	Mumbay	1.9	Lahore	2.7
11	Tianjin	1.8	Suzhou	2.6
12	Chongqing	1.8	Bangalore	2.6
13	Cairo	1.8	Baghdad	2.6
14	Manila	1.8	Ahmadabad	2.4
15	Shanghai	1.8	Hyderabad	2.4
16	Kolkata	1.7	Pune	2.3
17	Lima	1.4	Guangzhou	2.3
18	Shenzhen	1.1	Chennai	2.3
19	Istanbul	1.1	Delhi	2.3
20	Mexico	0.9	Ho Chi Minh	2.3
21	Buenos Aires	0.7	Hangzhou	2.2
22	London	0.7	Kuala Lumpur	2.2
23	Sao Paulo	0.7	Beijing	2.1
24	Rio de Janeiro	0.6	Yangon	2.1
25	Paris	0.6	Jakarta	2.0
26	Los Angeles	0.5	Chengdu	2.0
27	New York	0.5	Nanjing	2.0
28	Moscow	0.0	Mumbai	1.9
29	Osaka	-0.1	Xian	1.9
30	Tokyo	-0.1	Cairo	1.8

Notes: This table shows the fastest-growing megacities among: (1) the top 30 cities in 2015, and (2) the cities of at least 5 million inhabitants in 2015. *Sources:* Chandler (1987) and United Nations (2014).