August, 2020 Working Paper

ANATOMY OF DENSITY I:

Measurable Factors that Together Constitute Urban Density

+SHLOMO (SOLLY) ANGEL, PATRICK LAMSON-HALL, AND ZELTIA GONZALES BLANCO

ABSTRACT

We advance *Urban Density*—the ratio of the total population of a city and its overall urban footprint—as the single most useful metric for measuring the overall compactness of cities as well as for measuring progress in their densification, now a recognized sustainability objective. Yet we also realize that adopting *Urban Density* as a single measure of compactness hides more that it reveals. The key contribution of our study is in strengthening its usefulness by exposing its anatomy, showing that it can be factored into separate metrics. By factoring *Urban Density* into two, three, four, and then seven factors, we provide a structure for ordering most of the familiar density metrics used by planners, thus contributing to increased 'density literacy.' We show, for example, that *Urban Density = Floorspace Occupancy × Floor Area Ratio × Residential Share,* where *Floorspace Occupancy* is the average number of people per hectare of floor area in the city; the *Floor Area Ratio* is the ratio of the total residential floor space and the total residential area in the city; and *Residential Share* is the share of the residential area in the city's overall urban footprint. Our second contribution is a rigorous and replicable methodology for measuring *Urban Density* and all its factors in a representative group of ten cities from all world regions. Factoring *Urban Density* shows, for the first time, how different cities acquire their density: Hong Kong, for example, gets its high *Urban Density* from its high *Floor Area Ratio*; Kinshasa from its high *Floorspace Occupancy*; and Dhaka from its high *Floorspace Occupancy* and *Residential Share*. Our anatomy of density offers a novel outline for a comprehensive strategy for city densification, one that focuses on possibilities for increasing each of the seven factors that constitute *Urban Density* through selective policy interventions.

I Introduction: *Urban Density* as a Metric of Interest

1. *Urban Density* as a Key Sustainability Metric:

Increasing *Urban Density*—simply defined here as the ratio of the total number of inhabitants living within a well-defined footprint of a city and the total area of this footprint—has been correctly identified as a critical sustainability objective (see, e.g., Anderson, Kanaroglou, and Miller, 1996; Makido, Dhakal, and Yamagata, 2012; Kim and Brownstone, 2013; Lee and Lee, 2014; Seto et al., 2014; Ye *et al*., 2015; Wang *et al*., 2015; Gudipudi et al., 2016; OECD, 2018). First, because *Urban Density* as defined above is the intervening measure that translates population into land consumption: A city with a given population will occupy a smaller footprint—and will therefore need to convert less of the surrounding countryside to urban use—when its *Urban Density* is higher. Such a city can be said to be more *compact.* Second, because the inhabitants of a more compact city with a higher *Urban Density*—will be closer to each other, making movement from place to place more efficient.[1] In both senses, therefore, a more compact city will be a more sustainable city.

Increasing *Urban Density*, or *Densification,* is now "enshrined in land use planning policy in many countries" (Burton, 2001, 219) as well as in Goal 11 (Indicator 11.3.1) of the United Nations *Sustainable Development Goals* (United Nations 2016, 14). Most developed countries and many less-developed ones now pursue policies that promote compact urban form (OECD, 2012; OECD, 2018), and studies have indeed revealed a range of benefits from compactness: more productivity; lower cost public services; enhanced social and economic mobility as well as diversity; increased public transit use; lower energy use and emissions; and improved health and well-being (for a comprehensive review, see Boyco and Cooper, 2011).[2]

2. Confronting the Multiplicity of Density Measures:

Surely, *Urban Density* as defined above is by no means the only way to measure density in cities. Several authors (e.g. Alexander 1988, Churchman 1999, and Forsyth 2003) have written comprehensive reviews of a large number of urban density measures, noting that such measures have been applied at different scales—a single residential plot, a residential subdivision, a neighborhood, or an individual census tract—and not only to the city as a whole. Following these earlier authors, we classify the main density metrics found in the literature in the table below.[3] The abundance of singular definitions—coupled with the use of different density metrics in different studies—can be overwhelming for many urban planners and policy makers, making it difficult to compare densities in different cities or their change over time, not to mention to study their impact on desired sustainability and other outcomes. Given the multitude of definitions of density, Dovey and Pafka (2016) call for 'greater density literacy.'

Density Category	Ratio	Reference
Indoor Density		
Persons per Dwelling Unit	$\frac{Total\ Population}{Total\ No.of\ Dwelling\ Units}$	U.S. Census Bureau, 1950; Grebler, 1951;
Persons per Habitable Room	$\frac{Total\ Population}{Total\ No.of\ Rooms}$	Blake, Kellerson and Simic, 2007;
Floor Area per Person	$\frac{Total\ Area\ of\ Dwelling\ Units}{Total\ Population}$	Flood, 1996; Blake, Kellerson and Simic, 2007;
Occupied Floor Area per Person	$\frac{Total\ Occupied\ Floor\ Area}{Total\ Population}$	WHO, 2009
Parcel Density		
Floor Area Ratio	$\frac{Gross\ Residential\ Floor\ Area}{Residential\ Plot\ Area}$	ASPO, 1958; Brueckner *et al.*, 2016
Dwelling Unit per Hectare	$\frac{Total\ No.of\ Dwelling\ Units}{Residential\ Plot\ Area}$	Alexander, 1993; Forsyth, 2003
Residential Neighborhood Density		
People per Residential Neighborhood Area	$\frac{Total\ Population}{Residential\ Neighborhood\ Area}$	Eldridge, 1984; APHA, 1960
Dwelling Units per Residential Neighborhood Area	$\frac{Total\ No.of\ Dwelling\ Units}{Residential\ Neighborhood\ Area}$	APHA, 1960; Alexander, 1988; Forsyth, 2003;
Dwelling Units per "Developable Land" Area	$\frac{Total\ No.of\ Dwelling\ Units}{Total\ "Developable\ Land"\ Area}$	Galster *et al.*, 2001;
Citywide Density		
Citywide Floor Area Ratio	$\frac{Total\ Floor\ Area}{Area\ of\ Urban\ Footprint}$	McDonald and McMillen, 2010; Krehl *et al.*, 2016;
Citywide Floor Area per Person	$\frac{Total\ Population}{Total\ Floor\ Area}$	Krehl *et al.*, 2016;
Citywide People per Total Residential Area	$\frac{Total\ Population}{Total\ Residential\ Area}$	Frenkel and Ashkenazi, 2005;
Urban Density	$\frac{Total\ Population}{Area\ of\ Urban\ Footprint}$	James, 1967, 55; Angel *et al.*, 2016; OECD, 2018
Built-up Area Density	$\frac{Total\ Population}{Built-up\ Area\ in\ Urban\ Footprint}$	Angel *et al.*, 2016

Table 1: The main measures of urban density in the literature and selected sources that mention them.

The anatomy of density introduced in this study seeks to increase density literacy and go one step further. First, it brings together many of the disparate definitions of density and applies them all to a single spatial unit: in our case, to the city or metropolitan area as a whole.[4] Second, and more importantly, it gives them order and structure by focusing on the strict arithmetical relationships—mostly involving multiplication and division—between them. To the best of our knowledge, there are few writings that discuss the arithmetical relationships between different density metrics (e.g. Segal, 1965) although Alexander (1988, 7) notes that given some assumptions, density metrics can be derived from one another. Third, in the name of making density literacy comprehensive, we also introduce a number of metrics that are not commonly associated with the study and measurement of urban density—The *Occupancy Rate*, *Floor Plan Efficiency,* and *Residential Share* to be defined later—while making sure that all the density metrics are part and parcel of one single measure, *Urban Density*, and need to be considered together, as parts of a single whole.

3. Retaining *Urban Density* as a Single Metric:

While many measures of density throw light on important urban phenomena and have considerable merit in planning and policy discussions, a mission-oriented urban public policy that measures its success by pursuing a single goal with a single measure for the city at large is likely to be easier to formulate, garner support, and implement (Maret, 2002) than a policy that seeks to attain disparate goals requiring disparate measures or to optimize a composite index with implicit or explicit weights assigned to different measures (e.g. Galster et al., 2001; Ewing, Pendell and Chen, 2002).

That said, adopting *Urban Density* as a single metric for measuring progress in densification in cities is problematic, for it may hide more than it reveals. A city with a higher *Urban Density* may have attained its high density because of overcrowding, the packing of large households into small one-room dwelling units, for example. In this city, densification through further increases in overcrowding may be of questionable merit. In another city, high-rise 'towers in the park' that may not result in higher *Urban Density* if such towers displace large households in small apartments in buildings with high plot coverage. Another city with high-density residential neighborhoods may exhibit a low overall *Urban Density* because a large share of its urban footprint is devoted to industrial use. Finally, another city may be a ghost city, built at high floor-area ratios but unoccupied, thus having zero *Urban Density*. These examples show that densities are the result of different phenomena. Any serious densification policy has to attend to all of these different phenomena if it is to be effective.

The key research question posed here is this: How can we retain *Urban Density* as the single, most appropriate measure of density by exposing its internal structure in a way that accounts for many, if not most, of the different factors—many of them densities of one kind or another—that act together to create the overall density of cities? Our most important research result is our answer to this question. In the following section we demonstrate that many, if not most, of the different density-related attributes that matter for city densification are in fact a set of *measurable* factors that when multiplied together reconstitute *Urban Density*.

II The Theory: Factoring Urban Density into its Constituents

1. Factoring defined

Factoring is simply defined as breaking down a quantity into its constituents in such a way that multiplying them by each other yields that quantity. The number 12, for example, can be factored into two numbers, 3 and 4, that when multiplied together yield 12. The volume of a box can be factored into its width, length, and height that when multiplied together yield the volume of the box. And in the same way that the number 12 can be factored in different ways—2 × 6 = 12, 3 × 4 = 12, or 2 × 2 × 3 = 12—*Urban Density* can also be factored into its constituents in different ways. Our approach to the anatomy of density focuses on the average values for various density factors for the city or metropolitan area as a whole.[5] Factoring *Urban Density* reveals the anatomy of this whole, in the sense of exposing its components. This makes it possible to treat them one by one while considering their effect on *Urban Density* as a whole, much in the way that understanding the human body's anatomy makes it possible to treat its organs one by one, while still considering their effect on the body as a whole. What is more, factoring *Urban Density* exposes both the role of individual density metrics and the relationships among them, thus giving both order and structure to the discussion of urban density, a subject that has been typically shrouded in confusion because of the large number of competing definitions used to characterize it as shown in table 1 above.

2. *Urban Density* as a Product of Two Factors

We define *Urban Density* as the ratio of the *Total Population* inhabiting a given *Urban Footprint* and the total area of the *Urban Footprint*, where the boundary of the *Urban Footprint* is defined by the city's *Extrema Tectorum*—the edge of its contiguous built-up area— including the urbanized open space contained within that built-up area[6]:

$$(1) \qquad Urban\ Density = \frac{Total\ Population}{Area\ of\ Urban\ Footprint}$$

We begin our illustration of the anatomy of density with the simplest decomposition of *Urban Density* into two factors—defined as citywide average ratios—*Floorspace Occupancy* and *Floor Area Density. Floorspace Occupancy,* a not-so-familiar concept, is simply the reciprocal of a more familiar one, the average residential *Floor Area per Person* in the city

(Flood, 1996; Blake, Kellerson and Simic, 2007). *Floor Area* refers to gross residential floor area, including wall thicknesses and common areas[7]—in contrast with the *Living Area in Dwelling Units,* to be introduced later, which is the net floor area within dwellings:

$$(2) \qquad Floor\ Area\ per\ Person = \frac{Total\ Residential\ Floor\ Area}{Total\ Population}$$

$$(3) \qquad Floorspace\ Occupancy = \frac{1}{Floor\ Area\ per\ Person} = \frac{Total\ Population}{Total\ Residential\ Floor\ Area}$$

We use *Floorspace Occupancy* as a factor of *Urban Density* instead of using *Floor Area per Person* because *Urban Density* increases proportionally when it increases. In contrast, *Urban Density* decreases proportionally when *Floor Area per Person* increases.

The second factor, *Floor Area Density*—a variation on the *Floor Area Ratio*—is a commonly used metric in urban economics to characterize the spatial structure of the urban built environment, e.g. to describe its decline with distance from the city center (e.g. McDonald and McMillen, 2011, 132; Krehl *et al*., 2016, 10). We define it as follows, restricting it to residential floor area in the entire *Urban Footprint*:

$$(4) \qquad Floor\ Area\ Density = \frac{Total\ Residential\ Floor\ Area}{Area\ of\ Urban\ Footprint}$$

When we multiply these two factors together, *Total Residential Floor Area* cancels out, and we get *Urban Density*:

$$(5) \qquad Floorspace\ Occupancy \times Floor\ Area\ Density = \frac{Total\ Population}{Total\ Residential\ Floor\ Area} \times \frac{Total\ Residential\ Floor\ Area}{Area\ of\ Urban\ Footprint} = Urban\ Density$$

An increase in *Floor Area Density* will result in an increase in *Urban Density* only if *Floorspace Occupancy* increases or remains unchanged. In other words, a significant decrease in *Floorspace Occupancy*—say, by more affluent families living in larger apartments displacing larger, poorer ones living in smaller apartments—may overshadow the effect of a significant increase in *Floor Area Density* on overall *Urban Density. Floor Area Density* may increase while overall *Urban Density* decreases. In short, building more floor space does not necessarily result in densification. This simple insight exposes the value of the anatomy of density: It allows us to gain a better understanding of how *Urban Density* changes as one or more of its constituents change.

3. Urban Density as a Product of Three Factors

Introducing a simple refinement, we can see that *Floor Area Density* can be perceived as the product of two factors: The average residential *Floor Area Ratio (FAR)*—a metric typically used to regulate the allowable building volume on a given plot (e.g. ASPO, 1958; Kogo, Kaneko and Morisugi, 2010; Brueckner et al., 2016)—and *Residential Share,* a common metric used in quantifying urban land use plans (e.g. Perez, Avault and Vrabel, 2004; Keys, Wentz and Redman, 2007). The *Floor Area* in the *Floor Area Ratio* is the gross residential floor area defined earlier. A low value for *Residential Share* explains that devoting large portions of the *Urban Footprint* to other land uses—be they streets, commercial, industrial, or civic areas, or open space—lowers *Urban Density*. It also explains that low employment density lowers *Urban Density*. These two metrics are calculated for the city as a whole and defined as follows:

$$(6)\qquad \textit{Floor Area Ratio} = \frac{Total\ Residential\ Floor\ Area}{Total\ Area\ of\ Residential\ Plots}$$

$$(7)\qquad \textit{Residential Share} = \frac{Total\ Area\ of\ Residential\ Plots}{Total\ Area\ of\ the\ Urban\ Footprint}$$

The *Total Area of Residential Plots* is defined net of streets, public spaces, or civic facilities.[8] We can now decompose *Urban Density* into three factors, calculated as averages for the city as a whole: *Floorspace Occupancy* defined earlier, the *Floor Area Ratio*, and the *Residential Share*. The reader can easily ascertain that when we multiply these three factors together, everything cancels out, and we get *Urban Density*.

$$(8)\qquad \textit{Floorspace Occupancy} \times \textit{Floor Area Density} \times \textit{Residential Share} =$$

$$(9)\qquad \frac{Total\ Population}{Total\ Residential\ Floor\ Area} \times \frac{Total\ Residential\ Floor\ Area}{Total\ Area\ of\ Residential\ Plots} \times \frac{Total\ Area\ of\ Residential\ Plots}{Total\ Area\ of\ the\ Urban\ Footprint} = \textit{Urban Density.}$$

4. Urban Density as a Product of Four Factors

Introducing a further refinement, we can see that the *Floor Area Ratio* as defined here is the simple product of two familiar metrics, residential *Building Height* and *Plot Coverage,* calculated as averages for the city as a whole and defined as follows:

$$(10)\qquad \textit{Building Height} = \frac{Total\ Residential\ Floor\ Area}{Total\ Area\ of\ Residential\ Building\ Footprints}$$

$$(11)\qquad \textit{Plot Coverage} = \frac{Total\ Area\ of\ Residential\ Building\ Footprints}{Total\ Area\ of\ Residential\ Plots}$$

We measure *Building Height* as the average number of residential floors—exclusive of commercial floors in mixed-use buildings—in the city as a whole (e.g. ASPO, 1968; Bertaud and Brueckner, 2005). This way of measuring it assumes that identical residential floors—

with outlines identical to the building footprint under them—are stacked on top of one another. *Plot Coverage,* together with setbacks from the edges of plots, is often regulated to ensure that enough air and natural light is allowed to penetrate into rooms. It measures the average share of residential plots occupied by residential building footprints. Again, the reader can ascertain that when we multiply these two factors together, the *Total Area of Residential Building Footprints* cancels out and we get the *Floor Area Ratio*. This, in turn, means that we can represent *Urban Density* as a product of four factors:

$$(12) \quad \textit{Floorspace Occupancy} \times \textit{Building Height} \times \textit{Plot Coverage} \times \textit{Residential Share} = \textit{Urban Density}.$$

5. Urban Density as a Product of Seven Factors

Finally, we can decompose *Floorspace Occupancy* as defined earlier in Equation (1) into four factors to obtain an even more refined and more encompassing seven-factor anatomy of density. We define two of these four factors, *Dwelling Unit Occupancy* and the *Occupancy Rate,* as average values for the city as a whole calculated as follows:

$$(13) \quad \textit{Dwelling Unit Occupancy} = \frac{\textit{Total Population}}{\textit{Total Number of Occupied Dwelling Units}}$$

$$(14) \quad \textit{Occupancy Rate} = \frac{\textit{Total Number of Occupied Dwelling Units}}{\textit{Total Number of Dwelling Units}}$$

Dwelling Unit Occupancy is a measure of the average number of people occupying a single dwelling unit, typically one household, large or small, and sometimes doubled-up households. It is a more refined measure of overcrowding than *Floorspace Occupancy*, as it measures occupancy in the net floor area of occupied dwelling units, disregarding vacant ones (WHO, 2009?). *The Occupancy Rate*—and its complement, the *Vacancy Rate*—are common measures used in censuses to measure the utilization of the available housing stock (U.S. Census, 2019; Moreno and González Blanco, 2014). The third factor of *Floorspace Occupancy*, *Dwelling Unit Packing,* is the reciprocal of the more familiar *Dwelling Unit Size*, an average value defined as follows:

$$(15) \quad \textit{Dwelling Unit Size} = \frac{\textit{Total Living Area of Dwelling Units}}{\textit{Total Number of Dwelling Units}}$$

$$(16) \quad \textit{Dwelling Unit Packing} = \frac{1}{\textit{Dwelling Unit Size}} = \frac{\textit{Total Number of Dwelling Units}}{\textit{Total Area of Dwelling Units}}$$

Dwelling Unit Packing measures the number of dwelling units that can be fitted in a hectare of salable or rentable floorspace net of common areas like corridors, lobbies, staircases, or elevator shafts. We use it as a factor of *Urban Density* instead of using *Dwelling Unit Size* because, other things being equal, when it increases, *Urban Density* increases proportionally. In contrast, other things being equal, *Urban Density* decreases proportionally when the average *Dwelling Unit Size* increases.[9]

The fourth factor constituting *Floorspace Occupancy* is *Floor Plan Efficiency*. It is a measure commonly used by developers to calculate the salable floor space of their buildings, excluding wall thicknesses, common corridors and staircases, elevator and utility shafts, lobbies, common public areas and common open floors, storage areas, and areas dedicated to off-street parking (see, e.g., Barton, 2014; Humphries and Partners, 2019).

$$(17) \quad \textit{Floor Plan Efficiency} = \frac{\textit{Total Living Area of Dwelling Units}}{\textit{Total Residential Floor Area}}$$

The reader can ascertain that when we multiply these four factors together, everything cancels out and we get *Floorspace Occupancy.* We can then combine these four factors with three other factors defined earlier—*Building Height*, *Plot Coverage* and *Residential Share*—to produce an anatomy of density with seven factors. Again, the reader can verify that when we multiply these seven factors together, everything cancels out, and we get *Urban Density*:

$$(18) \quad \textit{Dwelling Unit Occupancy} \times \textit{Occupancy Rate} \times \textit{Dwelling Unit Packing} \times \textit{Floor Plan Efficiency} \times \textit{Building Height} \times \textit{Plot Coverage} \times \textit{Residential Share} = \textit{Urban Density.}$$

To summarize, *Urban Density* can be factored in a number of ways. The four decompositions of *Urban Density* into factors are summarized in figure 1 below.[10] Each decomposition of *Urban Density* into its factors exposes its anatomy and yields some important insight. In the four cases shown above we decomposed it into two, three, four, or seven important and well-established factors, most of which are commonly associated with density but remain hidden from view—and thus often disregarded and omitted from important policy discussions—when we focus too intensely on *Urban Density* alone. As we shall show in the concluding section of this article, exposing the anatomy of density has great value in helping formulate comprehensive densification strategies that seek to increase *Urban Density* by increasing each of its factors given the constraints—be they economic, political, or structural—pertaining to each factor.

Decomposing density into measurable factors also forces our estimates of each of these factors to conform to the arithmetical relationships among them—be they addition, subtraction, multiplication, or division—thus yielding more rigorous estimates. And while exposing the anatomy of density is the theoretical contribution of this paper, a secondary contribution reported in this paper is empirical; it is our attempt to demonstrate that the factors that constitute *Urban Density* are indeed measurable—and therefore well-defined, apt to be better understood, and therefore simpler to act on—by calculating average citywide values for each one of the factors introduced above in a representative sample of ten cities from ten different world regions. With the exception of Krehl *et al*'s pioneering paper (2016) that sought to measure *Floor Area per Person and Employee* and the average

Floor Area Ratio in four German cities, this, to our knowledge, has not been attempted before.

The results of our measurements are preliminary and quite possibly subject to some error.[11] Still, the errors appear to be small enough to allow us to illustrate two important insights. First, that factor values cannot be inferred from knowledge about *Urban Density*; in fact, several factors are not correlated with *Urban Density* at all, and even when they appear to be, outliers are common. Second, that there are important variations in the anatomy of density among cities: Different cities get their *Urban Density* from different combinations of factors. Given these preliminary results, we remain confident that with attainable increases in measurement accuracy, anyone can monitor changes in these factors in any city one-by-one, making it possible to track the contributions of specific policies targeted at individual factors to overall progress in densification.

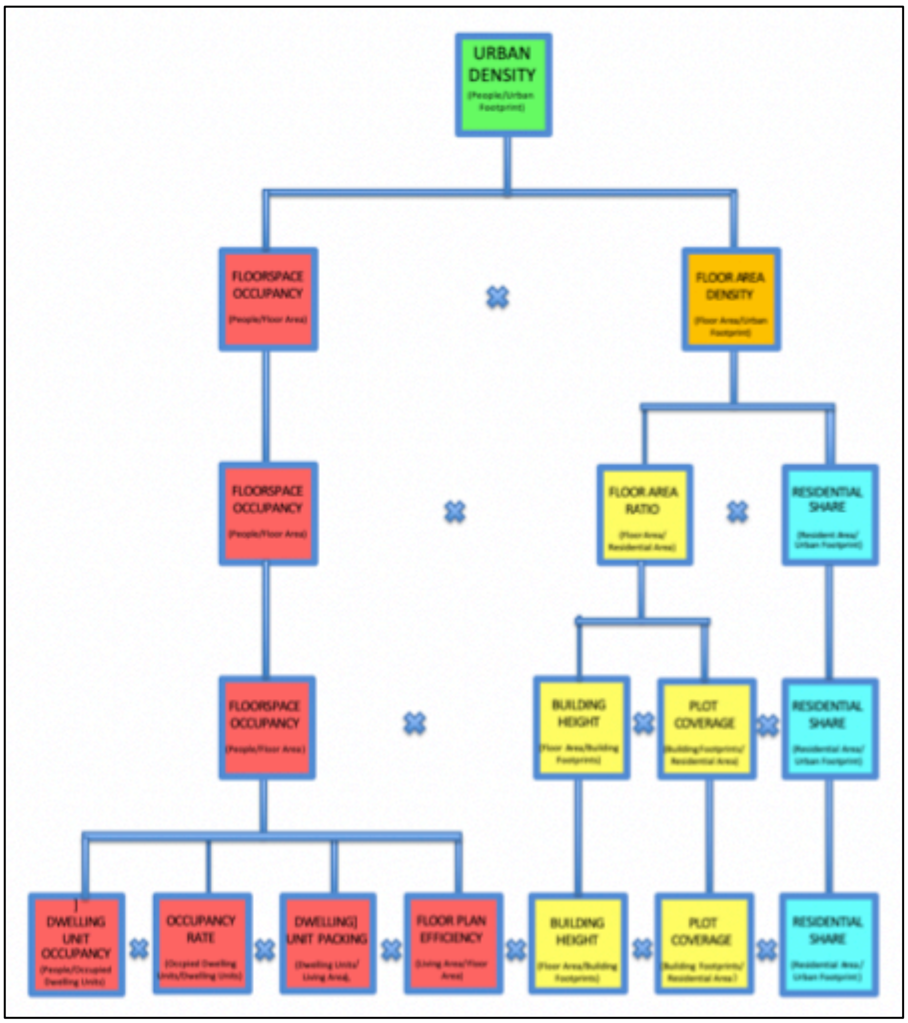


Figure 1: The two, three, four, and seven factors that, when multiplied together, constitute *Urban Density*.

III Methodology for Obtaining Empirical Results

1. Measuring Urban Density in Ten Representative Cities

To measure the factors that constitute *urban density* we selected ten representative cities (see figure 2) from the stratified global sample of 200 cities obtained from the universe of 4,231 cities and metropolitan areas that had 100,000 people or more in 2010 (Angel *et al*., 2016). The cities were Baku (Azerbaijan), Bangkok (Thailand), Bogotá (Colombia), Cairo (Egypt), Dhaka (Bangladesh), Hong Kong (Hong Kong, China), Kinshasa (Democratic Republic of Congo), Madrid (Spain), Minneapolis (Minnesota, United States), and Wuhan (China). They were selected from all major world regions.

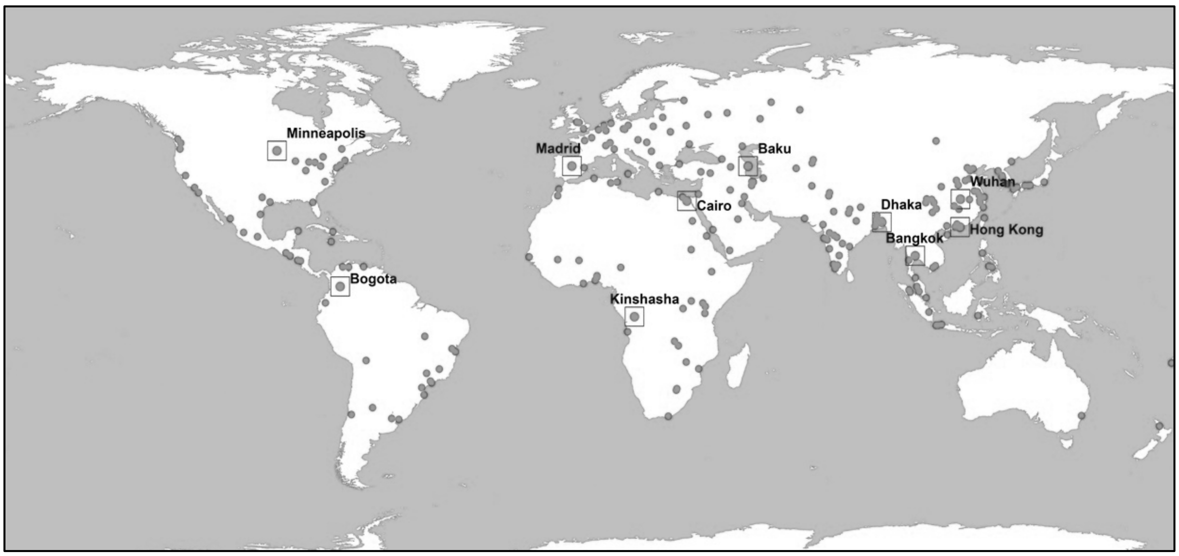


Figure 2: The locations of the 10 representative cities and the 200 cities in the global sample.

City	Country	Region	City GDP per Capita, 2012	Satellite Image Date	Urban Footprint at Date (hectares)	Population in Urban Footprint ('000)	Urban Density (p/ha)
Dhaka	Bangladesh	South and Central Asia	$ 4,979	1-Mar-14	36,541	13,609	372
Hong Kong	Hong Kong, China	East Asia and the Pacific	$50,746	1-Oct-13	12,278	4,322	352
Kinshasa	Congo Dem. Rep.	Sub-Saharan Africa	$ 1,849	1-Jul-13	45,681	10,226	224
Bogotá	Colombia	Latin America & the Caribbean	$15,933	1-Jan-10	39,723	7,802	196
Cairo	Egypt	North Africa	$12,067	1-May-13	136,396	15,735	115
Baku	Azerbaijan	Western Asia	$13,536	1-Aug-14	25,662	1,672	65
Madrid	Spain	Europe	$38,069	1-May-10	84,407	5,256	62
Bangkok	Thailand	Southeast Asia	$23,309	1-Jan-15	294,462	14,011	48
Wuhan	China	East Asia and the Pacific	$17,783	1-Sep-13	183,723	8,174	44
Minneapolis	United States	North America	$59,082	1-Oct-14	251,256	2,627	10

Table 2: Basic data on the ten representative cities, arranged by their *Urban Density*, from the highest (Dhaka) to the lowest (Minneapolis).

Table 2 above provides summary information on relevant characteristics of these cities. The table also shows that *Urban Density* in this group of cities was not highly correlated with City GDP or with city population.

To gain insight into the factors of U*rban Density* in each city, we needed to explore their anatomy of density. In this section we focus on a rigorous and replicable methodology for obtaining good estimates for eight, and only eight, metrics in each city that allow us to calculate *Urban Density* and all its factors. Those metrics are defined in Table 3 below:

Metric	Definition
Urban Footprint	The total area of the contiguous built-up area of the city and its urbanized open space
Total Population	The total population residing within the urban footprint
Residential Share	The share of the urban footprint occupied by residential buildings/plots
Plot Coverage	The share of the total area of residential plots occupied by residential buildings
Building Height	The average number of residential floors on a unit area of a residential building footprint
Floor Plan Efficiency	The average share of the gross residential floor area allocated to living areas in dwelling units
Occupancy Rate	The share of the total number of dwelling units that are occupied
Persons per Dwelling Unit	The average number of persons per dwelling unit in the city

Table 3: The eight metrics that need to be obtained in a city to calculate *Urban Density* and all its factors and their definitions.

Measuring the values for these eight metrics required empirical data of two kinds: data that could be obtained from the analysis of satellite imagery, and data that required secondary sources, essentially census data and architectural plans. Where possible, published reports were also consulted to confirm our estimates. In this section, we provide short summaries that explain how the values for each of these eight metrics were obtained. Detailed explanations are provided in the Technical Appendix.

2. The Urban Footprint

Several authors (e.g. Banai and DePriest, 2014) have noted that the administrative boundary of the city is not an appropriate denominator for measuring its density, because it may be too large or too small in comparison with its contiguous built-up area and may change arbitrarily from one year to another. We previously created *Urban Footprints* for the contiguous built-up areas of a global stratified sample of 200 cities including these ten cities (Angel *et al*., 2016) by classifying recent *Landsat* imagery in study areas containing these cities.[12] These footprints were used in the present study.

3. City population

We previously estimated the population living within the urban footprints of the global sample of cities, including the ten cities studied here, using digitized maps for census enumeration districts,[13] as well as estimates provided by the Chinese Academy of Sciences for Chinese cities. Using *Landsat* imagery of the urban footprint, population was apportioned among the census enumeration districts in the city.[14] *Total Population* of the city is the sum of the populations of all of the built-up pixels that fall within the city's urban footprint.

4. *Residential Share*

The measurement of *Residential Share* employed an intra-urban spatial sampling methodology that identified a set of quasi-random points at a desired point density based on a *Halton Sequence* (Halton, 1964). An image analyst determined for each point in the Halton sequence whether the land use at that point was 'residential' or 'non-residential'.[15] A running average value was calculated after the fact. In general, because we did not adopt an appropriate stopping rule, too many Halton points—more than 2,000—were sampled in each city. As a result, the running average value of the *Residential Share* stabilized in eight of the ten cities when considerably fewer points were sampled.

5. *Plot Coverage*

To measure *Plot Coverage,* the boundaries of areas surrounding the first few hundred sampled Halton points identified as 'residential' earlier were digitized. These boundaries could be streets surrounding residential city blocks or intra-block boundaries between 'residential' and 'non-residential' land uses. As with *Residential Share*, the bounded areas identified as 'residential' included non-residential buildings. The footprints of all residential buildings within a bounded area defined as 'residential' were digitized and their total area was calculated. We digitized an average of 460 residential blocks in each of the ten pilot cities. *Plot Coverage,* like *Residential Share,* was also calculated as a running average.

6. *Building Height*

We estimated *Building Height* in the ten pilot cities by counting residential floors in the nearest residential building to a Halton point identified as 'residential' earlier. The building was classified into one of three building types: (1) Single-family; (2) Non-core multi-family; and (3) Core multi-family, where core buildings were defined as having centralized elevator shafts and stairwells. Analysts then counted the stories of that building, excluding floors that were identifiable non-residential uses (such as stores or parking).[16] We obtained *Building Height* values for an average of 1,500 buildings in each of the ten pilot cities. Again, *Building Height* was also calculated as a running average

7. ***Floor Plan Efficiency***

Floor Plan Efficiency estimates the share of the gross floor area of residential buildings devoted to dwelling units, net of wall thicknesses, lobbies, elevator shafts, stairwells and mechanical spaces. In single-family homes, this share is simply the share of living areas net of wall thicknesses. In multi-family buildings, estimating this share required examining architectural floor plans for buildings of varying sizes and heights, based on the three-way typology identified earlier.[17] Data gathering for this metric was preliminary. Approximately fifty floor plans in total were collected and digitized. Other sources of data were consulted where possible. This value was measured with real accuracy in Minneapolis, for example, where high quality data on buildings was available. Using this value as a baseline, corrections were made to reduce the value for Baku—indicating thicker walls of Soviet-era buildings (Wright, 1975, 25)—and increase the value for Kinshasa (indicating thinner walls), based on personal observations by the authors and their colleagues. The value for Wuhan was based on the rule-of-thumb used by real estate brokers there.

8. ***Occupancy Rate***

The *Occupancy Rate* of residential units was estimated from publicly available data (Moreno and González Blanco, 2014). Three methods were used, reflecting different levels of data availability: (1) In Cairo, Madrid and Minneapolis national census data provided the *Occupancy Rate* directly; (2) In Bangkok, Bogota, Dhaka and Hong Kong the census provided 'households sharing the same housing unit' and 'total number of domestic households' for the city. The former was subtracted from the latter, yielding the number of occupied units (with the assumption that every household occupied a single unit); (3) In Baku, Kinshasa and Wuhan the same arithmetic was used as in (2), but total number of dwelling units was calculated by dividing the total square meters of residential floor space in the city by the average dwelling unit size, estimated by multiplying floor area per person and average household size.

9. ***Persons per Dwelling Unit***

The total population and the total number of dwelling units was directly available from national census data for the ten cities studied here. These two values allowed us to calculate the average number of persons per dwelling unit in the cities studied. In some countries it was only available inferentially. In Azerbaijan, for example, the census provided the total residential floorspace in the city, average square meters per person, and average household size, from which the total number of dwelling units was estimated. The urban extent of the cities used by the respective censuses to collect this information was similar, but did not usually correspond exactly, to the *Urban Footprint* of the city we identified. We adopted the empirical value for *Persons per Dwelling Unit* calculated for this urban extent to the *Urban Footprint* as defined by us.

IV Empirical Findings in Ten Representative Cities

1. Eight Metrics Obtained from Primary and Secondary Data

In the previous section we briefly reported on the methodology for obtaining primary and secondary data on eight metrics, each a value for the city as a whole. These values, summarized in table 4 below, show a considerable variation among the ten cities.

Metrics Obtained from Primary & Secondary Data	Dhaka	Hong Kong	Kinshasa	Bogotá	Cairo	Baku	Madrid	Bangkok	Wuhan	Minnea-polis
a. Population ('000)	13,609	4,322	10,226	7,802	15,735	1,672	5,256	14,011	8,174	2,627
b. Urban Footprint (hectares)	36,541	12,278	45,681	39,723	136,396	25,662	84,407	294,462	183,723	251,256
c. Building Height (stories)	2.5	20.5	1.1	2.8	4.4	2.6	3.4	1.9	5.8	1.4
d. Plot Coverage	53%	22%	20%	52%	43%	35%	26%	44%	32%	11%
e. Residential Share	37%	16%	46%	31%	26%	35%	19%	20%	14%	36%
f. Persons per Dwelling Unit	4.2	2.8	5.1	3.6	2.1	3.8	2.3	3.0	2.2	2.4
g. Occupancy Rate	97%	96%	99%	96%	66%	88%	88%	96%	77%	96%
h. Floorplan Efficiency	85%	75%	95%	87%	79%	67%	83%	89%	75%	90%

Table 4: Estimated values for eight metrics obtained from primary and secondary data for the ten representative cities. Cities are arranged from left to right in order of decreasing *Urban Density*.

These eight metrics allowed us to calculate *Urban Density* and all of its ten factors defined earlier in Section II. We performed these calculations in two stages. First, we calculated six intermediary values, shown in table 5 below. Then we used the values obtained in both tables 4 and 5 to calculate *Urban Density* and its factors.

2. Six Intermediary Metrics

Table 5 below displays the six intermediary values used for calculating *Urban Densit*y and its factors in the ten cities.

Intermediary Metrics (calculated)	Calcu-lation	Dhaka	Hong Kong	Kinshasa	Bogotá	Cairo	Baku	Madrid	Bangkok	Wuhan	Minnea-polis
i. Gross Residential Floor Area (hectares)	b×c×d×e	17,805	8,643	4,481	17,737	66,310	8,117	13,927	50,264	47,286	13,789
j. Residential Area (hectares)	b × e	13,536	1,955	20,812	12,170	35,335	9,102	15,846	59,786	24,911	90,671
k. Residential Building Footprints (hectares)	b × d × e	7,110	422	4,182	6,314	15,213	3,141	4,112	26,076	8,095	10,179
l. Dwelling Units ('000)	a ÷ f	3,215	1,527	2,009	2,154	7,405	441	2,321	4,687	3,689	1,077
m. Occupied Dwelling Units ('000)	l × g	3,128	1,467	1,989	2,064	4,920	389	2,045	4,481	2,822	1,029
n. Area of Dwelling Units (hectares)	.i × h	15,140	6,495	4,256	15,378	52,127	5,439	11,519	44,639	35,465	12,377

Table 5: Estimated values for six intermediary metrics, calculated for the ten representative cities from metrics obtained from primary and secondary data.

The table indicates how they were computed from the values in table 4 in the column labeled 'Calculation.' Table 5 gives us a sense of the orders of magnitude of these totals in different cities.

3. Calculated Estimates for *Urban Density* and All Its Factors

The values for *Urban Density* and all its factors described in Section II could now be calculated from tables 4 and 5 above.[18] Table 6 and the figures accompanying it below display our estimates for *Urban Density*, ten of its factors, and three familiar complementary metrics that are commonly encountered in discussions of urban density. The cities in the table and in the graphs below are all arranged in order of declining *Urban Density.* The column labeled 'Calculation' in table 6 indicates how the values were computed from the values in tables 4 and 5.

	Urban Density and Its Factors	Calcu-lation	Dhaka	Hong Kong	Kinshasa	Bogotá	Cairo	Baku	Madrid	Bangkok	Wuhan	Minnea-polis
o.	Urban Density (persons/hectare)	a ÷ b	372	352	224	196	115	65	62	48	44	10
p.	Floorspace Occupancy (persons/hectare)	a ÷ i	764	500	2,282	440	237	206	377	279	173	191
q.	Floor Area Density	.i ÷ b	0.5	0.7	0.1	0.4	0.5	0.3	0.2	0.2	0.3	0.1
r.	Floor Area Ratio	.i ÷ j	1.3	4.4	0.2	1.5	1.9	0.9	0.9	0.8	1.9	0.2
e.	Residential Share (percent)	e	37%	16%	46%	31%	26%	35%	19%	20%	14%	36%
c.	Building Height (number of stories)	c	2.5	20.5	1.1	2.8	4.4	2.6	3.4	1.9	5.8	1.4
d.	Plot Coverage (percent)	d	53%	22%	20%	52%	43%	35%	26%	44%	32%	11%
s.	Dwelling Unit Occupancy (persons/occupied D.U.)	a ÷ m	4.4	2.9	5.1	3.8	3.2	4.3	2.6	3.1	2.9	2.6
g.	Occupancy Rate (percent)	g	97%	96%	99%	96%	66%	88%	88%	96%	77%	96%
t.	Dwelling Unit Packing (D.U./hectare)	l ÷ n	212	235	472	140	142	81	201	105	104	87
h.	Floorplan Efficiency (percent)	h	85%	75%	95%	87%	79%	67%	83%	89%	75%	90%
Complementary Metrics												
u.	* Floor Area per Person (sq.m.) [Reciprocal of p]	1 ÷ p	13	20	4	23	42	49	26	36	58	52
v.	* Dwelling Unit Size (sq.m.) [Reciprocal of t]	1 ÷ t	47	43	21	71	70	123	50	95	96	115
w.	* Occupied Floor Area per Person (sq.m.)	v ÷ s	11	14	4	19	22	29	19	30	33	45

Table 6: Estimated values for *Urban Density* and its factors calculated for the ten representative cities from metrics obtained from data presented in tables X and Y above.

Empirical measurements of the factors of density illustrate the practical value of studying the anatomy of density. The key finding in table 6 is that knowing that *Urban Density* is higher in one city than in another does not allow us to infer whether its factors are also higher than in the other city. In general, we would expect cities with higher urban densities to have higher values for each of the factors than cities with lower densities, but the empirical results in table 6 show that this is quite often not the case. The most interesting insights offered by this empirical analysis are deviations from this expectation, the outliers that highlight more detailed knowledge about the anatomy of density in specific cities that could not be inferred from knowledge about their *Urban Density* alone.

4. Observed Variations in *Urban Density*, *Floorspace Occupancy, Floor Area Density,* and *Floor Area per Person.*

Urban Density can be represented as a product of two factors as we showed in Equation (4) earlier:

(4) Urban Density = Floorspace Occupancy × Floor Area Density.

The graphs showing the observed variations in *Urban Density*, *Floorspace Occupancy, Floor Area Density* and *Floor Area per Person* in the ten representative cities appear in figure 7 below.

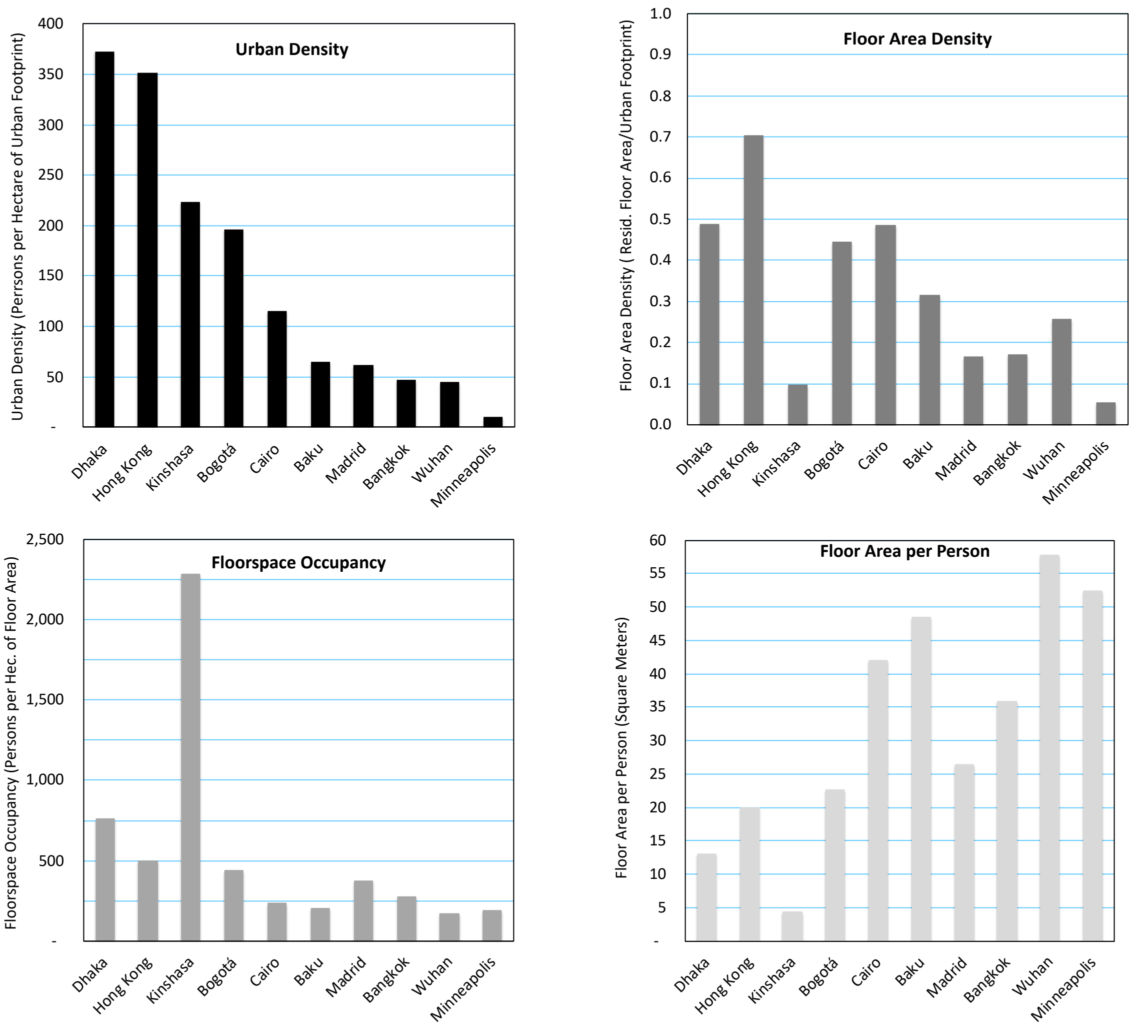


Figure 7: Observed variations in *Urban Density*, *Floorspace Occupancy, Floor Area Density,* and *Floor Area per Person* in the ten representative cities.

The variations shown in the graphs challenge a basic assumption: We would anticipate that cities with higher *Urban Density* would have lower *Floor Area per Person*, due to higher levels of crowding in residential areas. And *Urban Density* did tend to be higher in cities with higher *Floor Are Density* and *Floor Space Occupancy,* and lower in cities with higher

Floor Area per Person. However, *Floor Area Density* was higher in Hong Kong than in Dhaka, even though its overall *Urban Density* was lower; Cairo's *Floor Area Density* was equal to that of Dhaka even though its *Urban Density* was only the fifth highest in the group; and *Floorspace Occupancy* was highest in Kinshasa even though its *Urban Density* was only the third highest in the group. We thus begin to see that comparisons of *Urban Density* among cities sometimes hide more than they reveal about how its factors compare with one another.

5. Observed Variations in *Floor Area Ratio* and *Residential Share*

Equation (7) earlier showed *Urban Density* as a product of three factors:

(7) *Urban Density = Floorspace Occupancy × Floor Area Ratio × Residential Share.*

The graphs showing the observed variations in the *Floor Area Ratio* and the *Residential Share* in the ten representative cities appear in figure 8 below.

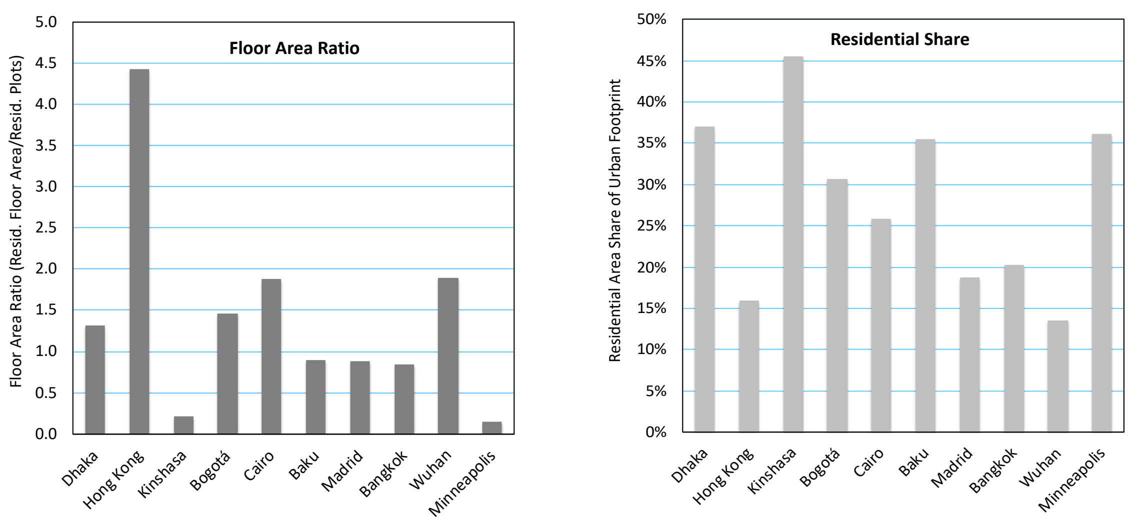


Figure 8: Observed variations in *Floor Area Ratio* and *Residential Share* in the ten representative cities.

Again, the empirical results challenge our core assumptions: We would expect cities with higher *Urban Density* to have a higher *Floor Area Ratio (FAR)* and a higher *Residential Share*; indeed, efforts to increase FAR are often a core part of efforts at densification. In fact, a visual inspection of figure 8 shows that neither factor is correlated with *Urban Density*. The three cities with the highest values for the *Floor Area Ratio*—Hong Kong, Cairo, and Wuhan—have the second-highest, fifth highest and second-lowest *Urban Density* in the ten-city group. The two cities with the lowest values for the *Residential Share*—Hong Kong and Wuhan—have the second highest and the second lowest *Urban Density* in the ten-city group.

Again, we see that values for the factors of *Urban Density* vary in ways that could not be predicted from information on *Urban Density* itself.

6. Representing *Urban Density* in Three Dimensions

Perceiving *Urban Density* as a product of three factors makes it possible to represent it as a box in three-dimensional space, where *Floorspace Occupancy*—measured on the X-axis—is its width, *Residential Share*—measured on the Y-axis—is its depth, and *Floor Area Ratio*—measured on the Z-axis—is its height. Representing *Urban Density* as a box for each of the ten representative cities—as shown in figure 9 below—begins to reveal its basic anatomy visually. The colored cube on the top left of the figure represents the ten-city averages and the dimensions of each box are simply multiples of these averages.

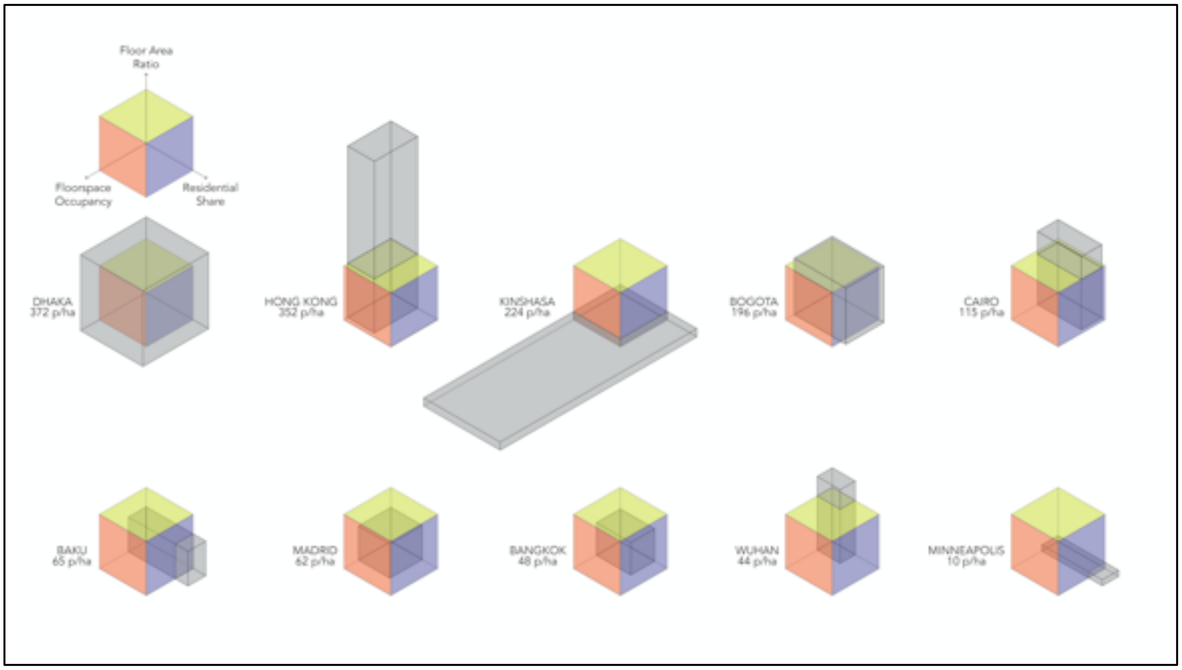


Figure 9: The urban densities of ten cities represented as volumes of boxes (in grey) in decreasing order from right to left and from top to bottom. The colored cube represents the 10-city average values for each of the three factors that make up *Urban Density.*

If each of the factors were perfectly correlated with *Urban Density*, we would expect all of the boxes to be cubes of decreasing size. In fact, we observe variations in the factors for each city. For example, Dhaka got its high density from its above-average *Floorspace Occupancy* and its above-average *Residential Coverage*, despite its *Floor Area Ratio* being below average. Hong Kong got its high density from its above-average *Floor Area Ratio*. Kinshasa got its high density from its above-average *Floorspace Occupancy*. Wuhan had a relatively low *Urban Density* despite its above-average *Floor Area Ratio*, largely because of its very low *Residential Share* and its below-average *Floorspace Occupancy*. All in all, figure 9 demonstrates that the fact that a city has a relatively high *Urban Density* does not necessarily imply that its factors all have above-average values, and that a city with a relatively low *Urban Density* does not necessarily imply that its factors all have below-average values.

7. Observed Variations in *Building Height* and *Plot Coverage*

Urban Density can also be represented as a product of four factors as we showed in Equation (10) earlier:

(10) *Urban Density = Floorspace Occupancy × Building Height × Plot Coverage × Residential Share.*

The graphs showing the observed variations in the *Building Height* and the *Plot Coverage* in the ten-city group appear in figure 10 below.

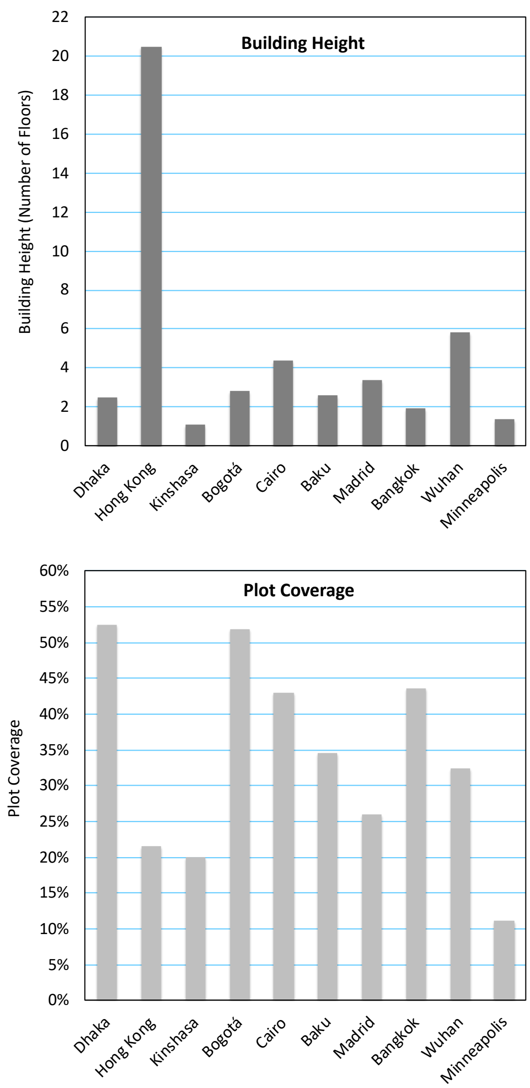


Figure 10: Observed variations in *Building Height* and *Plot Coverage* in the ten representative cities.

Building Height and *Plot Coverage* are the two factors that make up *Floor Area Ratio*, hence it comes as no surprise that the graphs show that they are uncorrelated with *Urban Density*. The three cities with the highest values for *Building Height*—Hong Kong, Cairo, and Wuhan—have the second-highest, fifth highest and second-lowest *Urban Density* in the ten-city group, for example. Kinshasa has a similar value for *Plot Coverage* as that of Hong Kong, while their respective *Building Height* values are at the two ends of the spectrum. Again, we see that values for the factors of *Urban Density* vary in ways that could not be predicted from information about *Urban Density* itself.

8. Observed variations in *Dwelling Unit Occupancy, Dwelling Unit Packing, Dwelling Unit Size, Occupancy Rate,* and *Floor Plan Efficiency*

Equation (17) earlier showed *Urban Density* as a product of seven factors:

(17) Urban Density = Dwelling Unit Occupancy × Occupancy Rate × Dwelling Unit Packing × Floor Plan Efficiency × Building Height × Plot Coverage × Residential Share.

The graphs showing the observed variations *Dwelling Unit Occupancy, Dwelling Unit Packing, Dwelling Unit Size, Occupied Floor Area per Person, Occupancy Rate,* and *Floor Plan Efficiency* in the ten representative cities appear in figure 11 below.

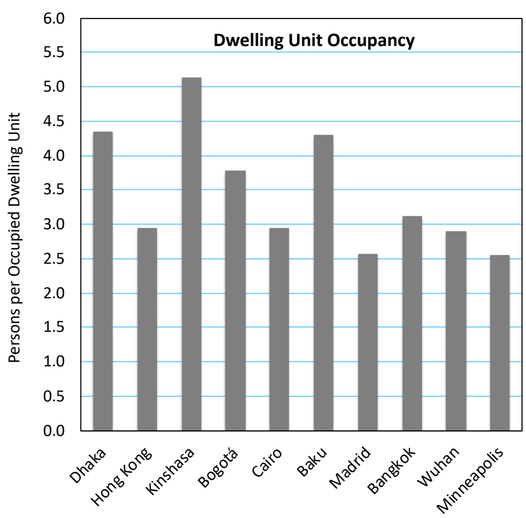
This factoring of *Urban Density* into seven distinct factors allows us to distinguish several cases in which our intuition about *Urban Density* is incorrect. For example, it appears that *Dwelling Unit Occupancy*—the average number of persons in an occupied dwelling unit—is not correlated with *Urban Density*. The values are lower, as expected, in cities in more-developed countries—Hong Kong, Madrid and Minneapolis—that all have slower birthrates and smaller household sizes.

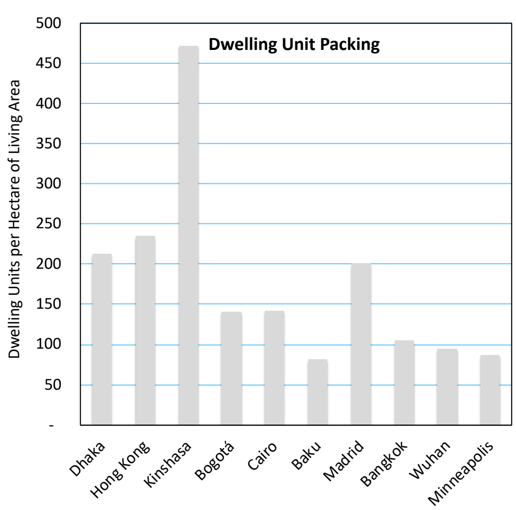
The *Occupancy Rate* in the ten-city group also does not appear to be correlated with *Urban Density* nor with any of the other factors. Cairo and Wuhan have notably low values. In Cairo, landlords' fear of not being able to evict tenants prevented them from renting their apartments, and in Wuhan a large share of the new housing stock remained vacant. *Floor Plan Efficiency* was also not correlated with *Urban Density* nor with any of its factors.

In contrast, we do find that *Dwelling Unit Packing* and its reciprocal *Dwelling Unit Size* are significantly correlated with *Urban Density*: Dwelling units are generally smaller in cities with higher density and therefore there are more dwelling units per hectare of residential floor area in these cities. When we translate *Dwelling Unit Size* to *Occupied Floor Area per Person,* the correlation with *Urban Density* becomes much stronger, but Kinshasa and Madrid remain outliers having lower than expected values.

To conclude, decomposing *Urban Density* into factors, while instructive in itself, gains in importance when we can obtain values for these factors for specific cities. The importance of these initial empirical values for the ten cities studied here is not their absolute values themselves but their ordinal relationship with density, which reveals—for the first time—that many of these factors are not correlated with *Urban Density* at all, and that their values cannot be inferred from the value of *Urban Density* alone. The anatomy of density in these cities reveals the relative strength of each factor, as well as the relationships between the factors, showing how they all act together to determine the overall *Urban Density* in each city.

What is more, decomposing *Urban Density* into a set of measurable factors alerts us to the real possibility of helping bring about the systematic densification of cities by attending to the possibilities of increasing the value of each and every one of these factors over time.[19] We discuss this insight in greater detail in the concluding section of this article.





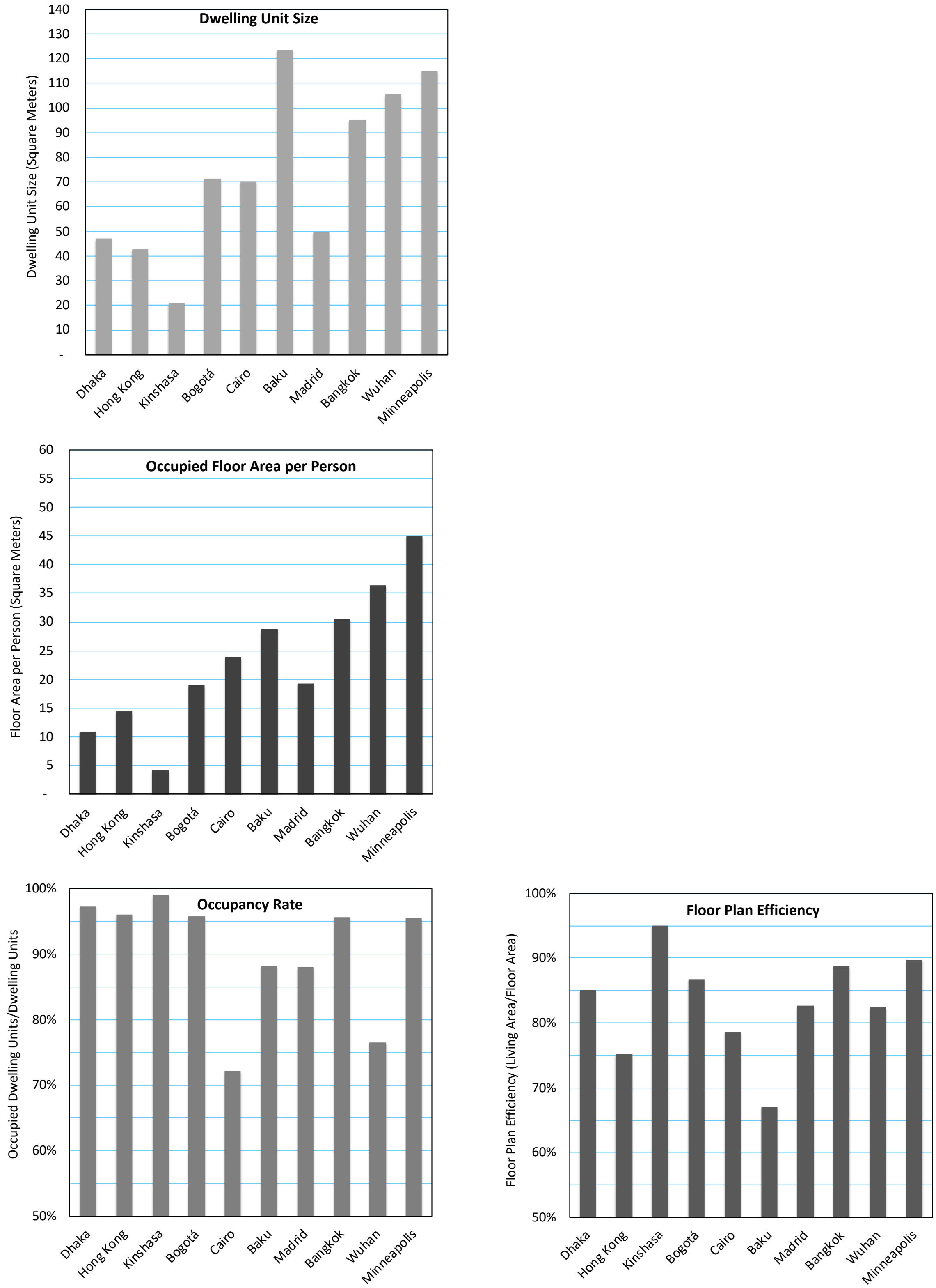


Figure 11: Observed variations in *Dwelling Unit Occupancy, Dwelling Unit Packing, Dwelling Unit Size, Occupied Floor Area per Person, Occupancy Rate,* and *Floor Plan Efficiency* in the ten representative cities.

V Conclusion: Towards a Comprehensive Strategy for Making Room for City Densification

In this paper we have laid out a new theory for decomposing *Urban Density* into measurable factors and a rigorous and replicable methodology for calculating these factors. This has made it possible to envision a comprehensive strategy for city densification based on increasing one or more of these factors, while monitoring the others to ensure that increases in some factors are not compromised by decreases in others. A concise outline of this strategy, based on decomposing *Urban Density* into seven factors, is summarized in table 11 as a checklist of possible interventions.

Increase Dwelling Unit Occupancy: ***Increase the number of people that occupy a single dwelling unit.***

- ***In times of crisis, enforce sharing of units by multiple households:*** At present, we find no effective ways for the public sector to increase dwelling unit occupancy except in times of emergency or crisis (Sosnovy, 1959).
- ***Encourage higher fertility rates:*** While outside the purview of municipal planners, this is often a national policy aimed at increasing household size (Howe, 1999)
- ***Encourage communal living:*** Remove financial and regulatory barriers to cohousing arrangements (Strauss, 2016).
- ***Decrease dwelling unit size:*** Decrease the size of dwelling units by increasing Dwelling Unit Packing (See below)

Increase the Occupancy Rate: ***Remove barriers and incentives that keep a larger-than-necessary share of the housing stock vacant.***

- ***Introduce a vacancy tax:*** Introduce a properly targeted vacancy tax that would encourage absentee owners to rent out their units, thus making better use of the city's housing stock (City of Vancouver, 2019).
- ***Permit leasing of vacant homes:*** Remove the restrictions imposed by building and neighborhood associations that forbid leasing of vacant homes and apartments (New York State Supreme Court, 1999).
- ***Overcome reluctance to rent vacant units:*** Remove the risk to rental-unit owners of not being able to repossess them (The World Bank, 2007).
- ***Avoid supply of units not in demand:*** Provide developers, investors, and speculators with reliable market information that can reduce the occurrence of ghost cities and an over-supply of unsold housing units (Shelton, Zhou and Pan, 2018; (OECD 2015).
- ***Reverse central city abandonment:*** Remove bureaucratic obstacles and provide fiscal and financial incentives to facilitate the renovation of abandoned residential properties (Furman Center for Real Estate and Urban Policy, 2006).

Table 11: A checklist of possible interventions to increase each one of the seven factors of Urban Density.

Increase Dwelling Unit Packing: ***Encourage people to reduce their consumption of floor area per person and to refrain from increasing it as they become better off.***

- ***Allow the subdivision of dwelling units***: Remove barriers to the subdivision of larger dwelling units into smaller ones, allowing the addition of kitchens and bathrooms (Bibby, Hennenberry and Halleux, 2018).
- ***Remove minimum size of apartments:*** Remove barriers to the minimum size of apartments and allow micro-apartments (Ville de Montréal 2015; Gabbe, 2015; Manville, 2013).
- ***Eliminate lending bias:*** Remove financial and fiscal incentives that give preference to single-family homes over apartments and to owner-occupied units over rental ones (Beyer, 2017).
- ***Avoid displacement through urban renewal:*** Avoid urban renewal that displaces smaller homes inhabited by larger households by larger homes with smaller ones (Wang *et al*., 2019; Wu *et al*., 2016).
- ***Encourage residential mobility:*** Remove regulatory and financial barriers to residential mobility that prevent older people from moving to smaller dwelling units (Sánchez and Andrews, 2011).

Increase Floor Plan Efficiency: ***Remove barriers and incentives that lower the share of living areas in the total residential floor area under construction.***

- ***Encourage lightweight construction:*** Support research and dissemination of building technologies that reduce the structural footprint in typical floor plans (Wright, 1971).
- ***Improve high-rise building design***: Increase awareness of Floor Plan Efficiency in the design of floor plans in high-rise buildings with a view to maximizing the share of salable living areas (Barton, 2014; Humphreys and Partners, 2019).
- ***Reduce parking requirements:*** Relax zoning regulations that mandate on-site parking space minimums and place maximums on allowable on-site parking spaces (Brinkman, 1948; Bertha, 1964; Fulton, 1999; Manville, Beata, and Shoup, 2013).

Increase Building Height: ***Revise regulatory barriers and incentives that prevent the increase of residential floor place with taller multi-story buildings.***

- ***Relax building height restrictions:*** Revise local zoning regulations that restrict the maximum height of buildings (Bertaud and Brueckner, 2005; Grabar, 2018).
- ***Allow adding floors on existing roofs:*** Relax restrictions that prevent the addition of floors to existing residential buildings (Berg, 2017).
- ***Increase access to construction finance:*** Remove barriers that block access of the residential sector to financial markets and prevent the construction of tall buildings (Renaud, 1987).
- ***Increase allowable Floor Area Ratios:*** Revise local zoning regulations that restrict the maximum allowable Floor Area Ratio (FAR) on residential plots (Common Floor, 2014; Shanker, 2018).
- ***Expand zoning for multi-family buildings:*** Increase the share of areas where multi-family buildings are permitted in residential zones (Harney, 2009; Mervosh, 2018; City of Minneapolis, 2018)

Increase Plot Coverage: ***Remove or redesign regulations that limit the share of the area of residential plots that building footprints can occupy.***

- ***Reduce minimum plot size for single-family homes:*** Allow single-family homes to be built on small plots, removing zoning restrictions that mandate large plot sizes (Chang, 2018).
- ***Relax setback regulations:*** Increase the share residential areas where no setbacks are required on the front and sides of plots and narrow setbacks are permitted in back (Price, 2017; Riis, 1890 reprinted 1971, 211)).
- ***Allow multiple units on single-family plots:*** Relax regulations that limit the construction of dwelling units on a residential plot to a single unit (Cohen, 2018; Garcia, 2017).
- ***Increase allowable Floor Area Ratios:*** Revise local zoning regulations that restrict the maximum allowable Floor Area Ratio (FAR) on residential plots (Common Floor, 2014; Shanker, 2018).

Increase Residential Share: ***Increase the share of residential and mixed-use areas in the city, while limiting the loss of residential areas to other land uses.***

- ***Accelerate conversion to residential use:*** Accelerate the conversion of brownfield sites, empty buildings, and underused lands held by public agencies to residential use (Burks, 1970; Kuang, 2012; SoHo Broadway Initiative, 2018; City of Joburg Property Company, 2019).
- ***Encourage mixed use in commercial areas:*** Facilitate the conversion of commercial areas into mixed-use areas with structures containing housing and offices above stores (Pyati, 2017; City of Los Angeles, 2019).
- ***Avoid mass evictions for urban highways:*** Avoid the construction of highway projects that cut through and destroy viable neighborhoods without proper rehousing of their residents (Thurz, 1966; Short, 2006).
- ***Avoid over-allocation of lands for industrial use:*** Remove the fiscal incentives that push municipalities to allocate too much land for industrial use (Advisory Commission on Regulatory Barriers to Affordable Housing, 1991, 2-3–2-4; Bertaud and Renaud, 1995).
- ***Encourage safe building on steeper slopes:*** Replace restrictions that prevent residential construction on steeper slopes with regulations that make such construction safe (Wordie, 2016; Buildings Department, 2018).
- ***Limit the evacuation of flood-prone areas:*** Limit the managed retreat from flood-prone areas, allowing in-place adaptation wherever possible (FEMA, 2014).
- ***Introduce a vacant land tax:*** Bring lands that are being held vacant for speculative purposes into active residential use by imposing an annual tax on such lands (Lam and Tsui, 1998; Amirtahmasebi *et al*., 2016; Vincent, 2019).

Table 11[continued]: A checklist of possible interventions to increase each one of the seven factors of Urban Density.

It should be noted that cataloguing these interventions does not necessarily mean that the authors are advocating for them.[20] Many of these interventions are context-dependent: They are appropriate and likely to be successful in some contexts but not in others. Where we have identified cities where proposed interventions have been implemented, be it successfully or unsuccessfully, we provide an appropriate reference.

We suggest that cities—and, where appropriate, higher levels of government as well—study their capacity for densification under existing conditions; set themselves attainable and measurable densification goals for each of the seven factors that constitute *urban density*; and then closely monitor changes in their overall density as well as in its factors. The choice of a subset of pragmatic and effective interventions in a given city must be done with an understanding of the potential of employing specific strategies; of the possible gain in overall *Urban Density* from changes in individual factors; of the cultural and political barriers to their application; of the difficulties in revising the regulatory framework; and of the budget available for implementation. These are unique to every city and cannot be determined in advance. Needless to say, monitoring changes in density factors will require further refinements of the methodology outlined in this article, a promising and valuable challenge for further research.

We are well aware that urban planners are experts in understanding the practical difficulties of densification. We are also well aware that densification efforts in cities the world over during the past two decades have had only little success, and that average city population densities in the world at large have been in significant decline during this period (Angel *et al.,* 2016, table 1). Yet we believe that this fresh look at the anatomy of density affords an opportunity to envision and energize new and more comprehensive strategies for urban densification. The anatomy of density presented here affords urban planners an outline of a manual for assessing the capacity of cities to densify under present conditions—e.g. under present land use and zoning regulations, current public opinion, fertility rates, and population projections—as well as for preparing pragmatic future plans for increasing this capacity. Assessing the feasibility of implementing these pragmatic plans in cities the world over should give the planning profession a realistic estimate of the extent to which the compact city agenda can play the critical role envisioned for it in mitigating global warming in due time, while keeping cities productive and inclusive. Such an assessment is now overdue.

* * *

References

Advisory Commission on Regulatory Barriers to Affordable Housing, 1991. "Not in My Backyard": Removing Barriers to Affordable Housing, Washington, D.C.: U.S. Department of Housing and Urban Development. Online at: https://www.huduser.gov/publications/pdf/notinmybackyard.pdf.

Ahlfeldt, G. and Pietrostefani, E., 2018. Demystifying compact urban growth: Evidence from 300 studies from across the world", *OECD Regional Development Working Papers*, 2018/03, OECD Publishing, Paris.

Alexander, E.R., 1993.Density Measures: A Review and Analysis, Journal of Architectural and Planning Research, 10(3), Autumn, 181-202. Online at: https://www.jstor.org/stable/43028746?seq=1#metadata_info_tab_contents

Alonso, W., 1968. Predicting Best with Imperfect Data, *Journal of the American Institute of Planners*, 34(4), 248-255.

American Society of Planning Officials (ASPO), 1958. Floor Area Ratio, Planning Advisory Service Information Report 111, June. Online at: https://www.planning.org/pas/reports/report111.htm

Anderson, W.P., Kanaroglou, P.S., & Miller, E.J., 1996. Urban form, energy and the environment: A review of issues, evidence and policy. *Urban Studies* 33(1): 7-35.

Angel and Lamson-Hall, 2020. Anatomy of Density II: A Comprehensive Strategy for Making Room for City Densification, Marron Institute of Urban Management Working Paper (forthcoming), New York University.

Angel, S, Blei, A. M., Lamson-Hall, P., Galarza Sanchez, N., Parent, J, and Civco, D.L., 2016. *Atlas of Urban Expansion—2016 Edition*. Cambridge, MA: Lincoln Institute of Land Policy.

Banai, R. and DePriest, T., 2014. Urban Sprawl: Definitions, Data, Methods of Measurement, and Environmental Consequences, *Journal of Sustainability Education* 7, December. Online at: http://www.jsedimensions.org/wordpress/wp-content/uploads/2014/12/Banai-DePriest-JSE-Vol-7-Dec-2014.pdf

Barton, J., 2014. The Economics of Building Tall, New London Architecture. Online at: http://legacy.newlondonarchitecture.org/docs/james_barton_-_aecom.pdf

Barton, J., 2014. The Economics of Building Tall, New London Architecture. Online at: https://www.newlondonarchitecture.org/docs/james_barton_-_aecom.pdf.

Berg, M., 2017. What is 'Tama 38'? Blogpost. Online at: https://360dira.com/what-is-tama-38/.

Bertaud A. and Brueckner, J.K., 2005. Analyzing building-height restrictions: predicted impacts and welfare costs. Regional Science and Urban Economics 35: 109–125. Online at: 10.1016/j.regsciurbeco.2004.02.004 CrossRefGoogle Scholar

Bertaud, Alain and Renaud, Bertrand, 1995. Cities Without Land Markets: Location and Land Use in the Socialist City, Policy Research Working Paper 1977, Washington DC: The World Bank, March. Online at:

http://documents.worldbank.org/curated/en/991331468757791794/pdf/multi-page.pdf.

Bertha, B., 1964. Appendix A, in W. Smith (Ed.), The Low Rise Speculative Apartment. Berkeley, CA: Institute of Urban and Regional Development.

Beyer, S., 2017. The Federal Housing Administration (Still) Insures Suburban Sprawl Over Urban Density, *Forbes Magazine*, 31 July. Online at: https://www.forbes.com/sites/scottbeyer/2017/07/31/the-federal-housing-administration-still-insures-suburban-sprawl-over-urban-density/#24d351047080

Bibby, P., Hennenberry, J., and Halleux, J-M., 2018. Under the radar? 'Soft' residential densification in England, 2001-2011, *Environment and Planning B: Urban Analytics and City Science*, ISSN 0265-8135, 1-17. Online at: http://eprints.whiterose.ac.uk/129769/

Blake, K.S., Kellerson, R.L. and Simic, A., 2007. *Measuring Overcrowding in Housing*, prepared for the U.S. Department of Housing and Urban Development Office of Policy Development and Research, September. Online at: https://www.huduser.gov/publications/pdf/measuring_overcrowding_in_hsg.pdf

Bleicher, H., 1892. *Statistiche Beschreibung Der Stadt Frankfurt Am Main Und Innen Bevolkerung*. Frankfurt Am Main: J.D. Sauerländer's Verlag.

Bourne, L. S., (2001). The urban sprawl debate: Myths, realities, and hidden agendas. *Plan Canada* 41(4), 26–30. Online at: https://utahforresponsiblegrowth.org/pdf/urban_sprawl_debate.pdf

Boyko, C.T. and Cooper, R., 2011. Clarifying and Re-conceptualizing Density, *Progress in Planning* 76, 1–61.

Brinkman, L., 1948. Offstreet parking. *Journal of the American Institute of Planners*, 14(4), 26–27.

Brownstone, D., & Golob, T. F., 2009. The impact of residential density on vehicle usage and energy consumption. Journal of Urban Economics, 65(1): 91–98.

Brueckner, J., Fu, S., Gu, Y. and Zhang, J., 2016. Measuring the Stringency of Land-Use Regulation: The Case of China's Building-Height Limits, MPRA Paper No. 70289, March, Munich: MPRA. Online at: https://mpra.ub.uni-muenchen.de/70289/

Buildings Department, 2018. Slope Safety. Hong Kong: Government of Hong Kong. Online at: https://www.bd.gov.hk/en/safety-inspection/slope-safety/index.html.

Bunting T., P. Filion, and H. Priston, 2002. Density Gradients in Canadian Metropolitan Regions, 1971–96: Differential Patterns of Central Area and Suburban Growth and Change. *Urban Studies* 39, 2531–2552.

Burks, E. C., 1970. Mayor Asks Aid to 'SoHo' Artists. *The New York Times*, 24 September.

Burton, E., 2001. The Compact City and Social Justice, a paper presented to the Housing Studies Association Spring Conference, Housing, Environment and Sustainability, University of York, April. Online at: http://www.china-up.com:8080/international/special/pdf/8.pdf

Cervero, R, Kockelman, K, (1997). Travel Demand and The 3dD: Density, Diversity, and Design. *Transportation Research Part D: Transport and Environment* 2(3): 199-219.

Chang, N., 2018. Planning the Houston Way, Part II: Special Minimum Lot Size, *Cite: The Architecture and Design Review of Houston*, 21 March. Online at: http://offcite.org/planning-the-houston-way-part-ii-special-minimum-lot-size-by-nunu-chang/

Churchman, A., 1999. Disentangling the Concept of Density, *Journal of Planning Literature* 13(4), 389-411.

City of Joburg Property Company, 2019. Projects Overview. Online at: http://www.jhbproperty.co.za/index.php/projects/projects-overview.

City of Los Angeles, 2019. Official City of Los Angeles Municipal Code, Chapter 1, Sections 12.10.5 and 12.11.5. Online at: http://library.amlegal.com/nxt/gateway.dll/California/lapz/municipalcodechapteriplanningandzoningco/chapterigeneralprovisionsandzoning/article2specificplanning-zoningcomprehen?f=templates$fn=default.htm$3.0$vid=amlegal:lapz_ca$anc=.

City of Minneapolis, 2018. *Minneapolis 2040*, online at: https://minneapolis2040.com/policies/access-to-housing/

City of Vancouver, 2019. Empty Homes Tax—City of Vancouver. Online at: https://vancouver.ca/home-property-development/empty-homes-tax.aspx.

Clark, C., 1951. Urban Population Densities, *Journal of the Royal Statistical Society: Series A (General)* 14(4): 490-496. Online at: https://www.jstor.org/stable/2981088?origin=crossref&seq=7#metadata_info_tab_contents

Cohen, J., 2018. California ADU Applications Skyrocket After Regulatory Reform, Next City. Online at: https://nextcity.org/daily/entry/california-adu-applications-skyrocket-after-regulatory-reform

Committee on the Hygiene of Housing, American Public Health Association (APHA), 1958. VI: Neighborhood Density: Coordination of Housing Elements, in *Planning the Neighborhood*, Chicago: Public Administration Service, 63-72. Online at: https://books.google.co.il/books?id=GQ5QAAAAMAAJ&pg=PA67&hl=iw&source=gbs_toc_r&cad=3#v=onepage&q&f=false

Common Floor, 2014. Floor space index in 8 major cities, 28 April. Online at: https://www.commonfloor.com/guide/floor-space-index-in-8-major-cities-41800.html.

Dol, K. and Haffner, M., eds., 2010. Housing Statistics in the European Union. OTB Research Institute for the Built Environment, Delft University of Technology, The Hague: Ministry of the Interior and Kingdom Relations, September. Online at: http://www.hofinet.org/upload_docs/UPLOADED_housing_statistics_in_the_european_union_2010.pdf

Dovic, K. and Pafka, E., 2016. Urban Density Matters—But What Does It Actually Mean? *City Metric*, July. Online at: https://www.citymetric.com/fabric/urban-density-matters-what-does-it-actually-mean-2118

Eldridge, N., 1984. Population density: the choice of gross or net residential area, *Urban Policy and Research* 2(1): 23-26, Online at: https://www.tandfonline.com/doi/abs/10.1080/08111148408522642

Ewing R., and R. Cervero, 2001. Travel and the Built Environment: A Synthesis. *Transportation Research Record* 1780: 87–114.

Ewing, R., Pendell, R. and Chen, D., 2002. *Measuring Sprawl and Its Impact*, Washington D.C.: Smart Growth America. Online at: https://www.smartgrowthamerica.org/app/legacy/documents/MeasuringSprawl.PDF

FEMA, 2014. *Homeowner's Guide to Retrofitting: Six Ways to Protect Your Home From Flooding*. Department of Homeland Security. FEMA P-312, 3rd edition.

Flood, J., 1996. Urban and Housing Indicators, *Urban Studies* 34(10), Special Issue: Habitat II, 1635-1665. Online at: https://www.jstor.org/stable/43083776

Forsyth, A., Oakes, J. M., Schmitz, K. H., & Hearst, M., 2007. Does Residential Density Increase Walking and Other Physical Activity? *Urban Studies*, 44(4): 679–697.

Forsyth, Ann, 2003. Measuring Density: Working Definitions for Residential Density and Building Intensity, Design Brief 8, Design Center for American Urban Landscape, July. Online at: http://annforsyth.net/wp-content/uploads/2018/05/db9.pdf

Frank, L. D., and Pivo, G., 1994. Impacts of mixed use and density on utilization of three modes of travel: Single-occupant vehicle, transit, and walking. *Transportation Research Record*, 1466: 44–52.

Frenkel, A. and Ashkenazi, M., 2008. Measuring Urban Sprawl: How Can We Deal with It? *Environment and Planning B: Planning and Design* 35: 56-79. Online at: https://www.researchgate.net/publication/23541654_Measuring_urban_sprawl_How_can_we_deal_with_it

Fulton, W. B., 1999. *Guide to California planning*. Ventura, CA: Solano Press.

Furman Center for Real Estate and Urban Policy, 2006. Housing Policy in New York City: A Brief History, Working Paper 06–01, April. http://furmancenter.org/files/publications/AHistoryofHousingPolicycombined0601_000.pdf.

Gabbe,C.J.,2015. Looking through the lens of size: Land use regulations and micro-apartments in San Francisco. Cityscape, 17(2).

Galster, G., Hanson, R. , Ratcliffe, M.R., Wolman, H., Coleman, S. and Freihage, J. 2001. Wrestling Sprawl to the Ground: Defining and Measuring an Elusive Concept, *Housing Policy Debate* 12 (4): 681-717. Online at: http://www.gulfofmaine.org/2/wp-content/uploads/2014/03/Wrestling-Sprawl-to-the-Ground-Defining-and-Measuring-an-Elusive-Concept.pdf

Gove, W.R., Hughes, M, and Galle, O.R., 1983. *Overcrowding in the Household: An Analysis of Determinants and Effects*. New York and London: Academic Press.

Grabar, H., 2018. Why Was California's Radical Housing Bill so Unpopular? Slate, April 20. Online at: https://slate.com/business/2018/04/why-sb-827-californias-radical-affordable-housing-bill-was-so-unpopular.html.

Grebler, L., 1951. The Housing Inventory: Analytic Concept and Quantitative Change, The *American Economic Review* (41(2), 555-568.

Gudipudi, R., Fluschnik, T., Ros, A. G. C., Walther, C., & Kropp, J.P., 2016. City density and CO^2 efficiency, *Energy Policy* 91: 352-361.

Halton, J. (1964), Algorithm 247: Radical-inverse quasi-random point sequence, *Communications of the Association for Computing Machinery*, 7: 701-702.

Harney, B., 2009. The Economics of Exclusionary Zoning and Affordable Housing, *Stetson Law Review* 38, 459-489. Online at: https://www.stetson.edu/law/lawreview/media/38-2harney-pdf.pdf

Holtzclaw J., R. Clear, H. Dittmar, D. Goldstein, and P. Haas, 2002. Location Efficiency: Neighborhood and Socio-Economic Characteristics Determine Auto Ownership and Use—Studies in Chicago, Los Angeles and San Francisco. Transportation Planning and Techn*ology* 25, 1–27. Online at:

Howe, N., 2019. Nations Labor to Raise Their Birthrates, *Forbes*, 29 March. Online at: https://www.forbes.com/sites/neilhowe/2019/03/29/nations-labor-to-raise-their-birthrates/#2ef7dd26707d

Humphreys and Partners, 2019. Signature Designs. Online at: https://humphreys.com/signature-designs/

James, J.R., 1967. Residential Densities and Housing Layouts. *Town and Country Planning* 35 (11), December, 552-561.

Keys, E., Wentz, E.A. and Redman, C.L., 2007. The Spatial Structure of Land Use from 1970–2000 in the Phoenix, Arizona, Metropolitan Area, *The Professional Geographer* 59(1): 131-147. Online at: https://www.researchgate.net/publication/227641626_The_Spatial_Structure_of_Land_Use_from_1970-2000_in_the_Phoenix_Arizona_Metropolitan_Area

Kim, J., & Brownstone, D., 2013. The impact of residential density on vehicle usage and fuel consumption: Evidence from national samples. *Energy Economics* 40, 196-206.

Kogo, T., Kaneko, T. and Morisugi, H., 2010. Necessity of Minimum Floor Area Ratio Regulation: A Second-Best Policy, *Annals of Regional Science* 44: 523-539. Online at: https://www.researchgate.net/publication/225799109_Necessity_of_minimum_floor_area_ratio_regulation_A_second-best_policy

Krehl, A., Siedentop, S., Taubenböck, H. and Wurm, M., 2005. A Comprehensive View on Urban Spatial Structure: Urban Density Patterns of German City Regions, *ISPRS International Journal of Geo-Information* 76, May. Online at: https://www.researchgate.net/publication/303550426_A_Comprehensive_View_on_Urban_Spatial_Structure_Urban_Density_Patterns_of_German_City_Regions

Kuang, W., 2012. Spatio-temporal patterns of intra-urban land use change in Beijing, China between 1984 and 2008, *Chinese Geographical Science* 22(2) 210-220. Online at: https://link.springer.com/article/10.1007/s11769-012-0529-x.

Lam, A. H. S. and Tsui, S. W., 1998. Policies and Mechanisms on Land Value Capture: Taiwan Case Study. Lincoln Institute of Land Policy Working Paper WP98AL1. Cambridge MA: Lincoln Institute of Land Policy.

Lee, S. and Lee, B., 2014. The influence of urban form on GHG emission in the US household sector, *Energy Policy,* 68: 534-549.

Makido, Y., Dhakal, S., & Yamagata, Y., 2012. Relationship between urban form and CO^2 emissions: Evidence from fifty Japanese cities. *Urban Climate* 2: 55-67.

Manville, M. 2013. Parking Requirements and Housing Development, *Journal of the American Planning Association* 79(1), 49-66. Online at: https://www.researchgate.net/publication/271931754_Parking_Requirements_and_Housing_Development.

Manville, M., Beata A. and Shoup, D., 2013. Turning Housing Into Driving: Parking Requirements and Density in Los Angeles and New York, *Housing Policy Debate* 23(2), 350-375. Online at: https://www.researchgate.net/publication/272123368_Turning_Housing_Into_Driving_Parking_Requirements_and_Density_in_Los_Angeles_and_New_York.

Maret, I., 2002. Understanding the Diversity of Sprawl and the Need for Targeted Policies, Paper Presented at the Congress on Environmental and Planning, Oxford, 23-27 September. 2002.

McDonald, J.F. and McMillen, D.P., 2010. *Urban Economics and Real Estate: Theory and Policy,* Second Edition, Hoboken, NJ: John Wiley and Sions, Inc.

Mervosh, S., 2018. Minneapolis, Tackling Housing Crisis and Inequity, Votes to End Single-Family Zoning. *The New York Times*, 13 December. Online at: https://www.nytimes.com/2018/12/13/us/minneapolis-single-family-zoning.html

Moreno, E.L., and González Blanco, Z., 2014. Ghost Cities and Empty Houses: Wasted Prosperity. *American International Journal of Social Science* 3(2): 207-216.

Næss P., 2005. Residential location affects travel behavior—but how and why? The case of Copenhagen metropolitan area. *Progress in Planning* 63, 167–257.

Næss P., 2005. Residential Location Affects Travel Behavior—But How and Why? The Case of Copenhagen Metropolitan Area. *Progress in Planning* 63: 167–257.

National Research Council, 2009. Driving and the Built Environment: The Effects of Compact Development on Motorized Travel, Energy Use and CO_2 Emissions. Washington, D.C.: The National Academies Press, , Online at: https://www.nap.edu/catalog/12747/driving-and-the-built-environment-the-effects-of-compact-development

New York State Supreme Court, 1999. Four Brothers Homes at Heartland Condominium II v. Gerbino, Jr., 262 AD 2d 279 – New York: Appellate Division, 2 September. Online at: https://scholar.google.com/scholar_case?case=14022744079093093267&q=Four+Brothers+Homes+at+Heartland+Condominium+II,+et.+al.,+v.+Gerbino&hl=en&as_sdt=2006

Norman, J., Maclean, H.L. and Kennedy, C.A., 2006. Comparing High and Low Residential Density: Life-Cycle Analysis of Energy Use and Greenhouse Gas Emissions, *Journal of Urban Planning and Development* 132(1), March, 10-21. Online at: https://pdfs.semanticscholar.org/9755/8f2bef4d8ca0f3713687cd63e9b4c9532c68.pdf

OECD, 2012. *Compact City Policies: A Comparative Assessment*. OECD Green Growth Studies, OECD Publishing.

OECD, 2015. OECD Urban Policy Reviews: Mexico 2015—Transforming Urban Policy and Housing Finance, OECD Publishing. Online at: https://www.oecd.org/gov/policy-highlights-mexico.pdf.

OECD, 2018. *Rethinking Urban Sprawl: Moving Toward Sustainable Cities*, Paris: OECD Publishing.

Perez, Y., Avault, J. and Vrabel, J., 2004. Residential Land Use in Boston, Research Department, Report No. 592, March Boston: Boston Redevelopment Authority. Online at: http://www.bostonplans.org/getattachment/62f2b1bf-c8b3-434b-89e4-ea8f5c8508b4

Price, A., 2017. The Problem with Setbacks. *Strong Towns*. Online at: https://www.strongtowns.org/journal/2017/10/23/the-problem-with-setbacks.

Pyati, A., 2017. Baltimore's New Zoning Hoped to Boost More Mixed-Use Development, *Urban Land Magazine*, Washington D.C.: Urban Land institute, 8 August. Online at: https://urbanland.uli.org/inside-uli/baltimores-new-zoning-hoped-ease-barriers-mixed-use-development/

Radnovich, C. and Woodworth, W., 2019. Oregon Legislature finally adjourns but not without political fireworks, *Salem Statesman Journal,* 30 June.

Renaud, B., 1987. Another Look at Housing Finance in Developing Countries, *Cities*, February, 28-34.

Riis, J., 1890. *How the Other Half Lives: Studies among the Tenements of New York*. New York: Charles Scribner's Sons. Reprinted 1971, New York: Dover Publications. Online at: https://www.bartleby.com/208/

Saelens B.E., J.F. Sallis, and L.D. Frank, 2003. Environmental correlates of walking and cycling: Findings from the transportation, urban design, and planning literatures. *Annals of Behavioral Medicine* 25, 80–91.

Sánchez A.C. and Andrews D., 2011. To Move or not to Move? What Drives Residential Mobility Rates in the OECD? OECD Economics Department Working Papers No. 846. https://doi.org/10.1787/5kghtc7kzx21-en.

Segal, W., 1965. The Use of Land in Relation to Building Height, Coverage and Housing Density, *Ekistics* 19(110), January, 66-70. Online at: https://www.jstor.org/stable/43623643

Seto K. C., S. Dhakal, A. Bigio, H. Blanco, G. C. Delgado, D. Dewar, L. Huang, A. Inaba, A. Kansal, S. Lwasa, J. E. McMahon, D. B. Müller, J. Murakami, H. Nagendra, and A. Ramaswami, 2014. "Chapter 12: Human Settlements, Infrastructure and Spatial Planning", in *Climate Change 2014: Mitigation of Climate Change*, Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, online at: http://pure.iiasa.ac.at/id/eprint/11114/1/ipcc_wg3_ar5_chapter12.pdf

Shankar, A., 2018. …FSI Guidelines, *The Hindu*, 9 September. Online at: https://www.thehindu.com/real-estate/tns-latest-fsi-guidelines/article25453685.ece

Shelton, T., Zhou, C., and Pan, N., 2018. China's eerie ghost cities a 'symptom' of the country's economic troubles and housing bubble, *The World*, Australian Broadcasting Corporation, 26 June. Online at: https://www.abc.net.au/news/2018-06-27/china-ghost-cities-show-growth-driven-by-debt/9912186.

SoHo Broadway Initiative, 2018. *SoHo Zoning Guidebook*, September. Online at: http://sohobroadway.org/wp-content/uploads/2018/09/SoHo-Guidebook_September-2018_Web.pdf

Sosnovy, T.,1959. The Soviet Housing Situation Today, *Soviet Studies*, 11(1): 1-21, https://www.jstor.org/stable/148885?seq=1#metadata_info_tab_contents

Strauss, I.E., 2016. The Hot New Millennial Housing Trend Is a Repeat of the Middle Ages, *The Atlantic*, 26 September. Online at: https://www.theatlantic.com/business/archive/2016/09/millennial-housing-communal-living-middle-ages/501467/

The Math Forum, 2003. Average of Ratios vs. Ratio of Averages. The Math Forum, 24 October.

The World Bank, 2007. Arab Republic Of Egypt: Analysis of Housing Supply Mechanisms, Final Note, Report No. 41180, February, Washington D.C.: The World Bank. Online at: https://openknowledge.worldbank.org/bitstream/handle/10986/7554/411800REVISED01inal002030701PUBLIC1.pdf?sequence=1&isAllowed=y

Thurz, D., 1966. Where Are They Now? A Study of the Impact of Relocation of Former Residents of Southwest Washington who served in a Demonstration Project, Health and Welfare Council of the National Capital Area, Washington D.C.: Government Printing Office.

U.S. Census Bureau, 1950. *U.S. Census of Population: 1950,* Volume III: Census Tract Statistics.

U.S. Census, 2019. Quarterly Residential Vacancies and Homeownership, Third Quarter 2019, Release Number: CB19-157, 29 October. Online at: https://www.census.gov/housing/hvs/files/currenthvspress.pdf

UN-Habitat, 2019. What is a City? Online at: https://oldweb.unhabitat.org/wp-content/uploads/2019/02/City-Definition-UN-Habitat.pdf

United Nations, 2016. SDG Indicators: Global indicator framework for the Sustainable Development Goals and targets of the 2030 Agenda for Sustainable Development, Department of Economic and Social Affairs, Statistics Division, A/RES/71/313.

Ville de Montreal, 2015. By-Law 03-096. Online at: http://ville.montreal.qc.ca/sel/sypre-consultation/afficherpdf?idDoc=23608&typeDoc=1

Vincent, J., 2019. Non-Glamorous Gains: The Pennsylvania Land Tax Experiment, *Strong Towns*, 6 March. Online at: https://www.strongtowns.org/journal/2019/3/6/non-glamorous-gains-the-pennsylvania-land-tax-experiment.

Wang, S., Fang, C., Wang, Y., Huang, Y., and Ma, H., 2015. Quantifying the relationship between urban development intensity and carbon dioxide emissions using a panel data analysis. *Ecological Indicators,* 49: 121-131.

Wang, X., Tong, D., Gao. J., and Chen, Y., 2019. The reshaping of land development density through rail transit: The stories of central areas vs. suburbs in Shenzhen, China, *Cities* 89, June, 35-45. Online at: https://www.researchgate.net/publication/330601104_The_reshaping_of_land_development_density_through_rail_transit_The_stories_of_central_areas_vs_suburbs_in_Shenzhen_China#pf7

Wordie, J., 2016. Barren rock: Hong Kong's hillsides weren't always so lush and green. Post Magazine. *South China Morning Post*, 21 May. Online at:

https://www.scmp.com/magazines/post-magazine/article/1947277/barren-rock-hong-kongs-hillsides-werent-always-so-lush-and.

World Health Organization (WHO), 2009. 'Overcrowding', in *Children's Environmental Health Indicators*, online at: https://www.who.int/ceh/indicators/overcrowding.pdf

Wright, J., 1975. Industrial Building in the Soviet Union, Building Research Station, U.S. Department of Commerce, Online at: https://www.govinfo.gov/content/pkg/GOVPUB-C13-7d84f08cc44cb9903107dd699b9bfa39/pdf/GOVPUB-C13-7d84f08cc44cb9903107dd699b9bfa39.pdf

Wright, J., 1975. Industrial Building in the Soviet Union, Building Research Station, U.S. Department of Commerce. Online at: https://www.govinfo.gov/content/pkg/GOVPUB-C13-7d84f08cc44cb9903107dd699b9bfa39/pdf/GOVPUB-C13-7d84f08cc44cb9903107dd699b9bfa39.pdf.

Wu, C., Ye, X., Ren, F., Wan, Y., Ning, P. and Du, Q., 2016. Spatial and Social Media Data Analytics of Housing Prices in Shenzhen, China, *PLoS One* 11(10), October. Online at: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5082690/

Ye, H., He, X. Y., Song, Y., Li, X., Zhang, G., Lin, T., & Xiao, L., 2015. A sustainable urban form: The challenges of compactness from the viewpoint of energy consumption and carbon emission. *Energy and Buildings, 9*3: 90-98.

Zhou, B.B., and K.M. Kockelman, 2008. Self-selection in home choice: Use of treatment effects in evaluating relationship between built environment and travel behavior. *Transportation Research Record: Journal of the Transportation Research Board 2077:* 54–6.

Anatomy of Density:
Measurable Factors that Together Constitute Urban Density
Technical Appendix[1]

1. Background

The research described in the companion paper divides *Urban Density* into seven factors that can be multiplied together to produce *Urban Density*. These seven factors are derived from eight metrics obtained from primary and secondary data sources, along with six intermediary metrics that are products or quotients of the eight primary metrics. This technical appendix reports on the methods used to measure each of the eight primary metrics in ten pilot cities. We believe that the feasibility and accuracy of the measurement of these metrics on a global scale is a key consideration for the broadening of the global analysis of density that is proposed in this paper.

The eight primary metrics are:

Metric	***Brief Definition***
Urban Footprint	The total area of the city
Total Population	The total population residing within the urban footprint
Residential Share	The share of the urban footprint occupied by residential plots
Plot Coverage	The share of the total area of residential plots occupied by residential buildings
Building Height	The average number of residential floors on a unit area of a residential building footprint
Floor Plan Efficiency	The average share of the living area allocated to dwelling units in a unit of gross residential floor area
Occupancy Rate	The share of the total number of dwelling units that are occupied
Persons per Dwelling Unit	The average number of persons per dwelling unit in the city

Table A1: Describing the eight primary metrics used in the calculation of the factors of *Urban Density*.

These eight metrics described in this paper are obtained from primary and secondary sources. The first metric, *Urban Footprint*, provides the study area and unit of analysis for the measurement of the remaining seven metrics. Each city—including cities that are contiguous collections of individual municipalities grouped into a single metropolitan area—is regarded as having a single urban footprint. The values of the eight metrics obtained from primary and secondary data sources, the six intermediary metrics, and the seven factors that constitute *Urban Density* are all averages—expressed as single values—pertaining to the entire urban footprint of the city.

---

[1] With contributions from Manuel Madrid, Suman Kumar, and Sharhad Shingade.

2. The Ten Pilot Cities

The ten pilot cities featured in this paper were selected from the global sample of 200 cities developed by New York University and UN Habitat. This sample, stratified by region, city size and number of cities in country, was drawn from the universe of 4,231 cities with populations of 100,000 or more. This universe of cities was identified by New York University as an initial input for the creation of the Atlas of Urban Expansion: 2016 Edition. Population information and locations for these 4,231 cities came from three main sources of data: United Nations Population Division, citypopulation.de, and the Chinese Academy of Sciences which provided specific information on the universe of Chinese cities. The ten pilot cities were chosen with an eye to broad geographic representation and with a strong preference for a maximum diversity of circumstances, in preparation for employing this methodology in the entire sample of cities. The selected pilot cities were Baku (Azerbaijan), Bangkok (Thailand), Bogotá (Colombia), Cairo (Egypt), Dhaka (Bangladesh), Hong Kong (Hong Kong, China), Kinshasa (Democratic Republic of Congo), Madrid (Spain), Minneapolis (Minnesota, United States), and Wuhan (China) (Figure A1).

Figure A1: The locations of the 10 representative cities and the 200 cities in the global sample.

Table A2 below provides summary information on relevant characteristics of these cities. It shows that *Urban Density* in these cities varied from a maximum of 372 persons per hectare in Dhaka to a minimum of 10 persons per hectare in Minneapolis, a 37-fold difference. The table also shows that *Urban Density* in this group of cities is not highly correlated with City GDP or with city population.

City	Country	Region	City GDP per Capita, 2012	Satellite Image Date	Urban Footprint at Date (hectares)	Population in Urban Footprint ('000)	Urban Density (persons per hectare)
Dhaka	Bangladesh	South and Central Asia	$ 4,979	1-Mar-14	36,541	13,609	372
Hong Kong	China	East Asia and the Pacific	$ 50,746	1-Oct-13	12,278	4,322	352
Kinshasa	Congo Dem. Rep.	Sub-Saharan Africa	$ 1,849	1-Jul-13	45,681	10,226	224
Bogotá	Colombia	Latin America and the Caribbean	$ 15,933	1-Jan-10	39,723	7,802	196
Cairo	Egypt	North Africa	$ 12,067	1-May-13	136,396	15,735	115
Baku	Azerbaijan	Western Asia	$ 13,536	1-Aug-14	25,662	1,672	65
Madrid	Spain	Europe	$ 38,069	1-May-10	84,407	5,256	62
Bangkok	Thailand	Southeast Asia	$ 23,309	1-Jan-15	294,462	14,011	48
Wuhan	China	East Asia and the Pacific	$ 17,783	1-Sep-13	183,723	8,174	44
Minneapolis	United States	North America	$ 59,082	1-Oct-14	251,256	2,627	10

Table A2: Basic data on the ten representative cities, arranged by their *Urban Density*, from the highest (Dhaka) to the lowest (Minneapolis).

The development of the methodology for measuring the eight metrics that enable the calculation of the factors of *Urban Density* began with the publication of the *Atlas of Urban Expansion* (Angel *et al.,* 2012) in 2012. The *Atlas* used *Landsat* satellite imagery to measure the urban footprints of 120 cities in two time periods, 1990 and 2000, and mapped the expansion of a subset of 30 cities over a 200-year period using both satellite imagery and historical maps (Figure A1). That effort yielded valuable insights into global trends in urban expansion and urban densities.

The eight metrics are the necessary input data that enable the calculation the seven factors of density in the ten pilot cities. To remind the reader, this step is preceded by the calculation of six intermediary metrics as described in the companion paper. This technical appendix will not address the calculation of the six intermediary metrics nor the calculation of the factors of density themselves, both of which subjects are addressed in detail in the companion paper. Instead, it will focus on the methodology used to compute the values of the eight primary metrics, gathered from primary and secondary data sources.

3. Data Sources and Halton Points

The analysis of medium-resolution *Landsat* imagery described in the *urban footprint* methodology facilitates the measurement of the *Urban Footprint* of the city. The remaining metrics use the urban footprint as their area of analysis (because of the need to focus on a consistent area in order to calculate the factors of density with any accuracy), but they also include the use of other data sources. *City Population* relies on census data, which can be taken as the only authoritative source of localized population information. Three of the eight primary metrics are measured within the *urban footprint* using satellite imagery, but compliment the *Landsat* imagery with freely available high-resolution satellite imagery: *Residential Share, Plot Coverage,* and *Building Height.*

The use of satellite imagery is advantageous compared to other methods for several reasons. First, it improves data reliability. The collection and analysis of the imagery itself uses the same data sources and the same techniques in all cities in all world regions. Comparable satellite inputs and outputs can be obtained for cities that are separated geographically and that employ different political systems. Additionally, satellite imagery offers global coverage, which other data sources do not. Finally, the use of satellite imagery is also quicker and less costly than alternative data gathering approaches, which would likely consist of manual data gathering from municipal sources.

Bing imagery is freely provided by Microsoft and is available to the general public. The use of *Bing* imagery differs substantially from the use of *Landsat* imagery. *Landsat* imagery analysis relies on the use of digital data that comprises the image—pixels, which are processed by a computer program (ERDAS) under the supervision of an analyst. *Bing* imagery is a pixelated photograph, not a digital satellite image. Its analysis builds on an earlier method developed by the authors of the UN-Habitat report "Streets as Public Spaces and Drivers of Urban Prosperity" (UN-Habitat, 2013). Following the tradition of aerial imagery analysis, *Bing* imagery is used as a tracing background for the digitization and measurement of specific urban features such as roadways, city blocks, and building footprints (Figure A2). The Urban Expansion Observatory conducted this digitization and its subsequent analysis for this paper.

Estimates for three additional primary metrics require the collection of information that comes solely from secondary and tertiary data sources. Those factors are: *Floor Plan Efficiency*, the *Occupancy Rate*, and *Persons per Dwelling Unit.* The data sources for these metrics may include decennial censuses, architectural plans, and (rarely) data from real estate sources as well. Estimates can sometimes be directly obtained from the census, but often requires intermediate calculations. The secondary and tertiary sources used for the measurement of *Floor Plan Efficiency*, *Occupancy Rate*, and *Persons per Dwelling Unit* provide information that cannot be obtained remotely, but must be collected on the ground, through surveys such as censuses, and in some cases—in the search for building floor plans—on the World Wide Web.

Reliance on secondary data necessarily introduces a degree of imprecision into our estimates. To minimize the risk of relying on a single source, we sought to triangulate our initial findings from numerous data sources including public data, private data from firms or non-profit organizations, and published academic work such as graduate thesis papers. Where numbers have contradicted each other, we continued our search until the resulting values converged on a value that was supported by multiple sources. Still, our pilot effort to obtain values for all eight primary metrics of *Urban Density* for all the ten cities we studied should be considered exploratory and in need of further improvement.

Figure A2: The measurement of the width of a road using Bing satellite imagery as a tracing background.

The primary metrics listed above are averages for the city as a whole, and must thus rely on citywide information on all residential areas and all built residential structures. But digitizing all residential blocks, building footprints, and building heights by hand is too time consuming, rendering the prospect of replicating this analysis for several cities essentially impossible. A key innovation in the development of this work, therefore, was the invention by the authors of the *Atlas of Urban Expansion—2016 Edition* of an intra-urban spatial sampling methodology for the metrics that rely on *Bing* imagery - *Residential Share, Plot Coverage,* and *Building Height*. The purpose of the sampling framework was to introduce stochastic variability to the analytic process by measuring particular attributes of the urban fabric in a set of random locations within a city's urban footprint. The values of each attribute for these random locations are each treated as a separate observation in a stochastic process, and are then used to calculate average values for that attribute for the city as a whole.

The spatial sampling framework used to measure the factors of *Urban Density* was based on a statistical tool known as a *Halton Sequence*. In essence, a Halton sequence generates a quasi-random set of point coordinates within a given urban footprint that are more evenly distributed than a set of truly random points (see figure A3). Given the same point of origin as a starting point—e.g. a point to the Southwest of the city's urban footprint with a latitude and longitude measured in precisely one-tenth of a degree—a Halton Sequence will always generate the same set of points. Given a desired density of points within a given urban footprint, the number of points that a Halton Sequence generates can be easily determined. And since the points are generated one by one, their order is known and we can proceed starting with the first one and continue until we have picked the desired number of points to meet our accuracy goals. Figure A4 shows the set of 2,377 Halton points used to sample the heights of buildings within the urban footprint of Bangkok, Thailand. This set is a subset of the 9,496 Halton points that were used to determine

Residential Share in this city, a set that is too dense to display in a map of the size shown here.

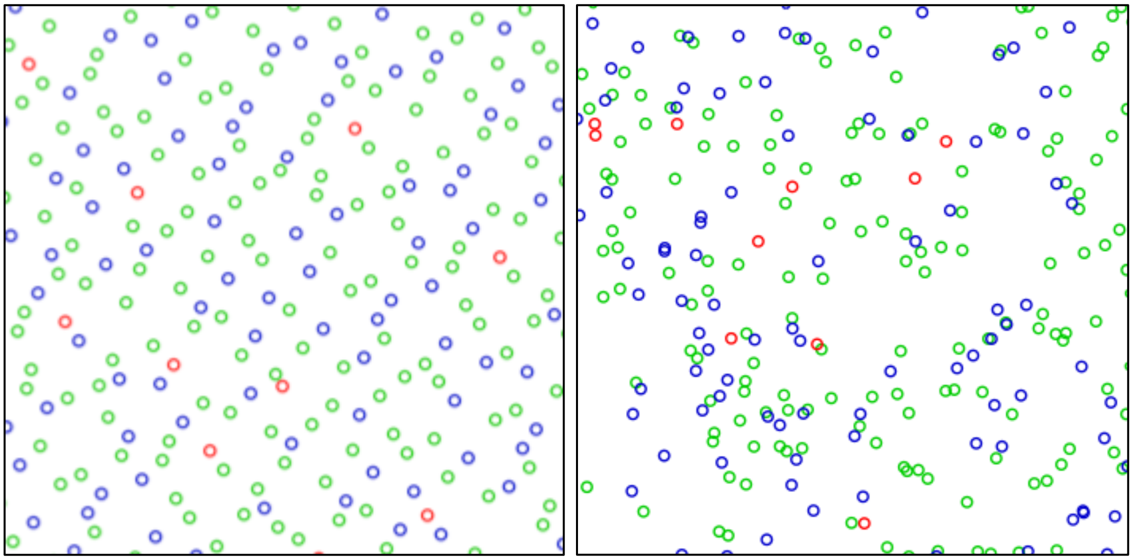


Figure A3: Quasi-random points generated by a Halton Sequence (left), compared to points generated by random coordinate numbers (right).

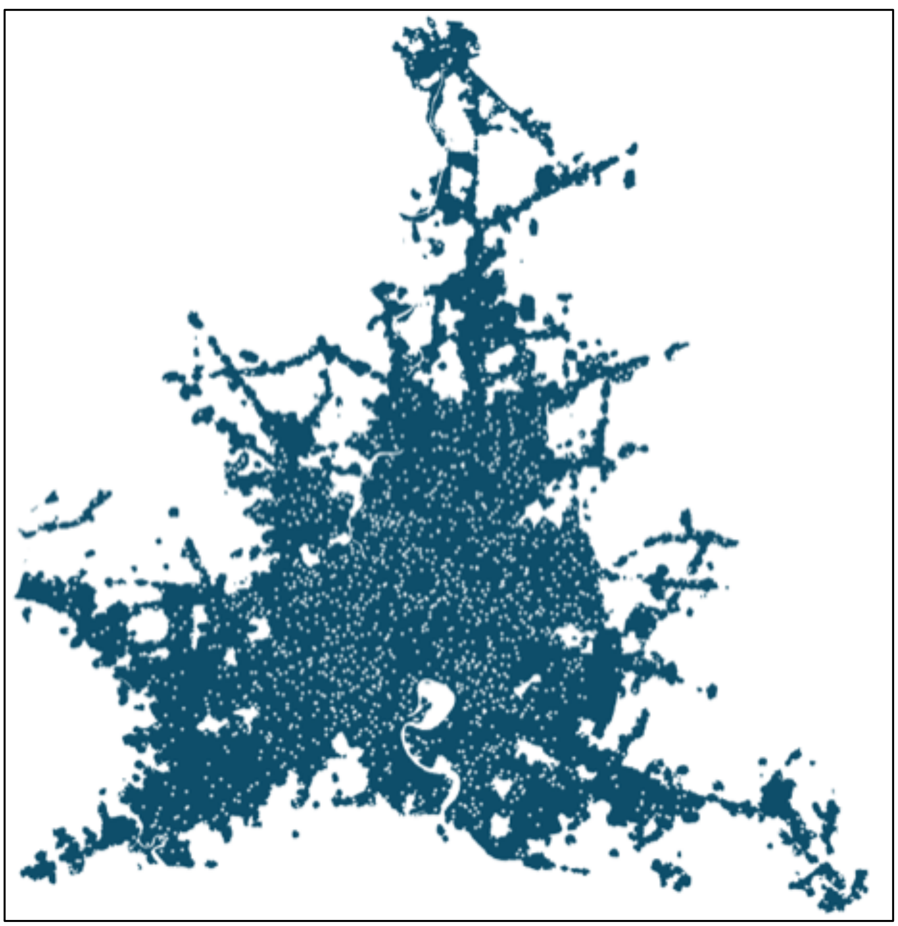


Figure A4: The set of 2,377 Halton points used to sample the heights of residential buildings located within the urban footprint of Bangkok, Thailand.

4. City Averages and Stopping Rules

The sampling strategy used to gather data in this paper involved manually examining thousands of points in ten different cities. Over the course of two years of research, our team continuously attempted to make the process more economical without sacrificing rigor. It became obvious by the end of our work that the methodology could be made yet more economical if a clear determination could be made about when to *stop* gathering additional samples. For this, we developed a stopping rule which is described in the following example:

An 'average' ratio of two quantities—for example, the average *Building Height* in a city, where *Building Height* is the ratio of *Floor Area* and *Building Footprints*—can have two distinct meanings (The Math Forum, 2003). The first is the *Average Building Height* in the city. It is a ratio of the total *Floor Area* in the city and the total area of *Building Footprints* in the city. The second is the *average (or mean) height of individual buildings in the city*. It is important to keep in mind that the two averages are not equal to each other: The first is obtained from a ratio of sums while the second is obtained from a sum of ratios. In this paper, we refer to the first average, the ratio of two sums, as the average value for the city as a whole. All the factors that constitute *Urban Density* are indeed ratios of sums, not mean values of individual observations, with the exception of *Building Height*, which is taken as mean values of individual observations.

For example, to obtain a reliable estimate of the average *Building Height,* we sample individual residential buildings in the city using a Halton Sequence, as illustrated above. For each building, we obtain a value for its height, measured in the number of residential stories. Many residential buildings also include other uses, such as commercial or industrial spaces or parking levels. In this case, the analyst identifies the floors that are residential, and only those are included in the estimate of building height. We also categorize the building into one of three types: 1) single family; 2) having a core that contains elevators and stairwells; 3) non-core. The categorization becomes relevant when calculating the city-wide value for *Floorplan Efficiency*, which requires an estimate of the total number of buildings in each category. The estimated average *Building Height* for the city can be approximated by the following procedure:

(1) For each building in the city, measure the number of residential floors.

(2) Add the number of floors of the first building in the sample (reflecting its position as the first residential building to be digitized in the Halton sequence) to the number of floors in the second building in the sample to obtain a cumulative number of floors;

(3) Calculate the estimate of the *Building Height* in the city by dividing the cumulative number of floors by the number of sampled buildings;

(4) Continue to add values for more buildings to the cumulative building height, and divide by the number of sampled buildings to get a cumulative average building height for the city;

(5) The resulting *Building Height* will gradually converge to an estimated value for the city as a whole.

(6) Employ a stopping rule: For example, stop adding buildings when the 95% confidence interval for the last 20 cumulative *Building Height* estimates falls below an agreed-upon value, say below 0.5% of the average *Building Height* for these 20 observations.

The results of this procedure for estimating *Plot Coverage*, *Residential Share*, and *Building Height* for the ten pilot cities are presented in graphic terms in the following sections. In calculating values for the ten pilot cities, we did not follow a strict stopping rule and, in fact, used different stopping rules for different cities, basing the number of sampled units simply on the areas of their urban footprints. As a result, in most cases we collected too much information. In calculating the density factors in the future, the number of sampled units could be drastically reduced.

5. Measuring the *Urban Footprint*

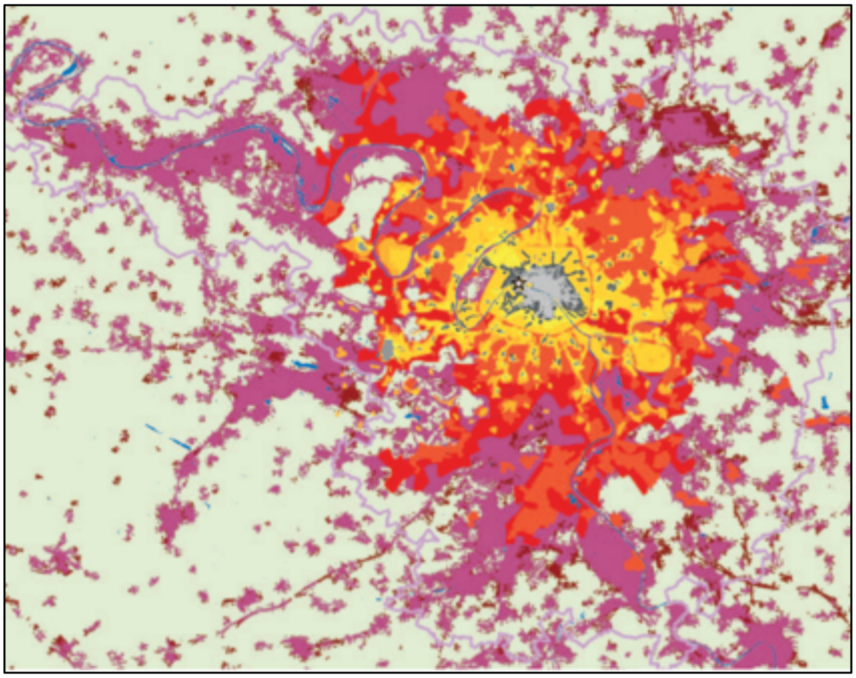


Figure A6: The expansion of Paris, France, 1800-2000, from the *Atlas of Urban Expansion—2012 Edition.*

The *Atlas of Urban Expansion—2012 Edition* introduced the concept of the *urban extent* as a means of measuring city area (Figure A6, for an example of the growth of an urban extent over time). In the Anatomy of Densities research it is known by an accompanying term, *urban footprint.* By this definition the edge of the city is identified as the edge of its continuous built-up area, inclusive of captured open spaces and fringe open spaces, which the ancient Romans used to refer to as its *Extrema Tectorum*.

An example of the unprocessed *Landsat* image can be seen in Figure A7. The U.S. National Aeronautics and Space Administration (NASA) collects and publishes this medium-resolution imagery at frequent intervals on a global scale. The information in each image, given free of charge, is expressed in 30-meter-by-30-meter pixels. This resolution is quite coarse by modern standards, but using *Landsat* imagery is preferable when longitudinal analysis is required because *Landsat* has an archive dating back to the 1970s. The classification and processing of this imagery is undertaken using a technique initially developed in 2011 in collaboration with the Department of Natural Resources and the Environment of the University of Connecticut. This procedure was updated in 2015 for the creation of the *Atlas of Urban Expansion: 2016 Edition.* The work of analyzing satellite imagery and generating urban footprints for cities is currently undertaken by the *Urban Expansion Observatory*, a research and analysis facility in Mumbai, India, established jointly by New York University, UN Habitat, and the Lincoln Institute of Land Policy and now managed by New York University and Valectus Ltd.

Figure A7: An unprocessed LANDSAT image showing logged land near a reservoir in Goin, Quebec in 1988.

The process of identifying a city's urban footprint proceeds as follows:

1) *Landsat* satellite imagery is classified using ERDAS software into *built-up* pixels, *open space* pixels, and *water* pixels (Figure A8). The study area is identified using Google Earth imagery and census boundaries from the national census, in most cases provided to us by CIESIN, but sometimes gathered from the census agency directly. The objective of the study area identification process is to select census boundaries that encompass the entire eventual urban extent of the city, meaning the urban extent should be completely surrounded by open space. This is eyeballed based on the visible built-up area of the city in Google Earth. The selected boundaries are then checked against the final urban extent, and the study area is increased if the urban extent is not completely surrounded by open space pixels. Exceptions to this rule are large contiguous metropolitan areas – specifically New York/Philadelphia/Hartford, Guangzhou/Shenzhen/Hong Kong, and Tijuana/San Diego. In New York, the New York and Philadelphia MSAs provided by the U.S. Census Bureau were used to divide the two metropolitan areas. In Guangzhou and in Tijuana the division was necessitated by the administrative break between the two conjoined cities, although they do function as unified economic areas.

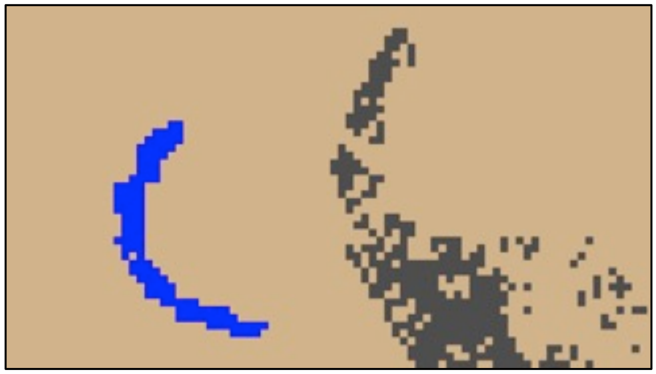


Figure A8: 30 x 30 meter pixels in freely available *Landsat* imagery can be classified into built-up, open space, and water areas.

2) Each pixel identified as built-up is further characterized as *urban built-up*, *suburban built-up*, or *rural built-up*. This characterization is based on the proximity of a given pixel to other built-up pixels. To measure this, a circle with an area of 1km^2—the *Walking Distance Circle*—is constructed around each pixel, and the built-up pixels within that radius are counted. Those built-up pixels having Walking Distance Circles with the majority of the pixels within them classified as built-up are marked as *urban* pixels. Those pixels having 25-50 percent of the pixels within their Walking Distance Circle classified as built-up are marked as *suburban* pixels (Figure A9). All other built-up pixels are considered *rural* pixels.

3) Pixels that are touching, including diagonal connections, are considered to be in clusters. Clusters of *urban* and *suburban* pixels are given a buffer of *fringe open space* pixels within 100 meters of each of these urban and suburban pixels. Rural built-up pixels that fall within this 100m buffer are also included in the cluster.

4) Areas of *captured open space* that are completely surrounded by urban and suburban pixels, and are less than 200 hectares in area, are identified as belonging to these urban and suburban clusters and are included.

5) The primary urban cluster of a given city is identified using a point with the latitude and longitude of the Central Business District (CBD), typically captured as the City Hall, or the central police station. These CBD points are manually verified. This primary urban cluster is also typically the largest urban cluster within the study area. This cluster and all other clusters in the study area are then surrounded with a buffer with an area equal to one-quarter of their areas. The urban clusters whose buffers overlap with that of the largest urban cluster, together with the largest cluster, are united to form the *Urban Footprint* of a given city.

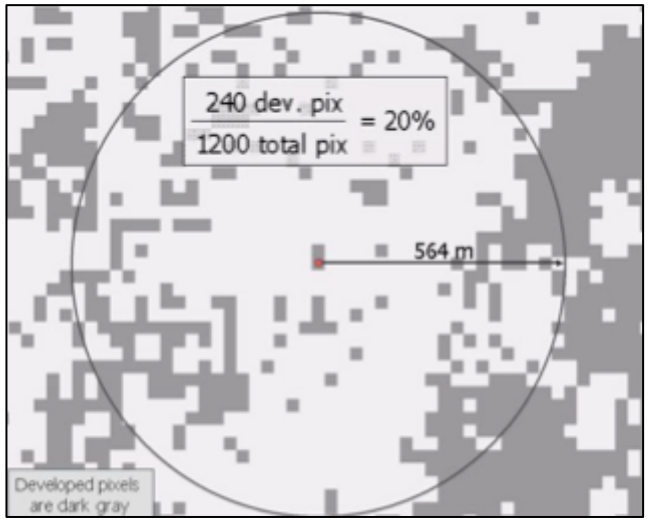


Figure A9: The pixel identified by the red dot is classified as a *rural* pixel because only 20 percent of the pixels within its Walking Distance Circle the circle are classified as built-up.

This method of identifying a city's urban footprint based on built-up area is preferable to other methods that rely, for example, on administrative boundaries alone, or on density cutoffs. Administrative boundaries are necessary for accessing social and economic information about a city, but they do not necessarily correspond to the observed built-up areas of cities and can change at a moment's notice, making them unsuitable for measuring and comparing urban density. For example, Figure A10 compares the administrative areas of Metro Manila, Philippines, and Minneapolis, Minnesota, United States. Metro Manila's administrative area is more than three times larger than that of Minneapolis. Figure A11 compares the actual urban footprints of these two cities in 2015. Minneapolis's urban footprint is more than double that of Metro Manila's.

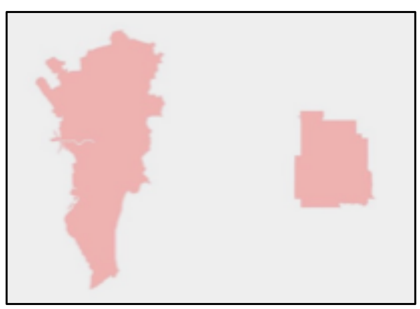


Figure A10: The administrative area of Metro Manila (left) is four times the administrative area of Minneapolis (right), shown here at the same scale.

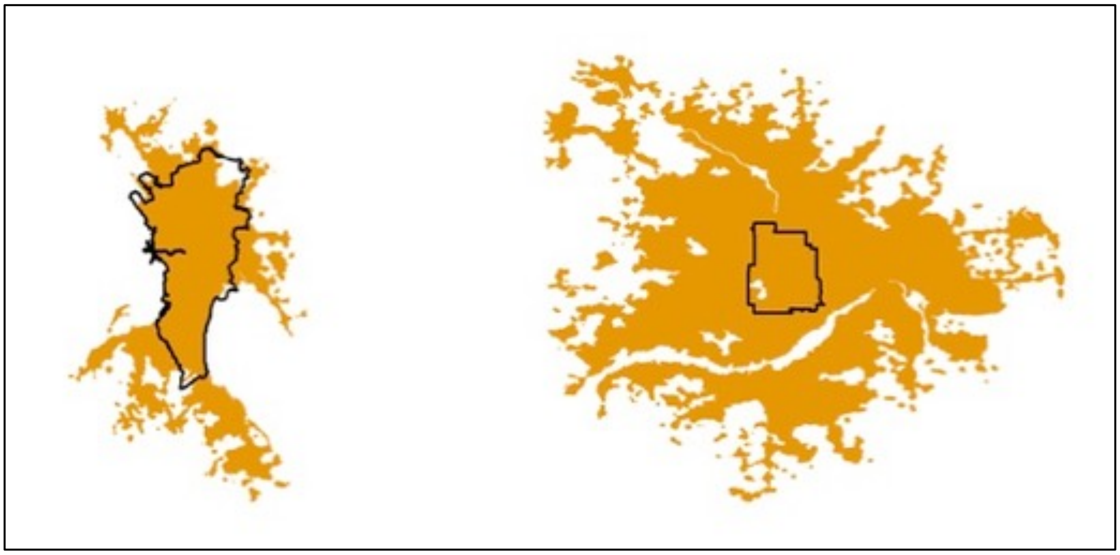


Figure A11: The *Urban footprint* of Metro Manila (left) is less than half that of Minneapolis (right), shown here at the same scale.

Universal density cutoffs to distinguish urban from rural areas yield unsatisfactory results as well. Densities on the suburban fringe of U.S. cities, for example, are higher than average densities in the rural areas of several countries, e.g. Bangladesh or Indonesia. The *Atlas of the Human Planet 2016*, for example, (Pesaresi et al, 2016, 23) has adopted a density cutoff of 300 persons per km^2 for 'urban clusters,' presumably so as to accommodate American suburbs. This has resulted, as we have pointed out elsewhere (Angel et al., 2018), in classifying too much of the planet's built-up areas as 'urban': *Atlas of the Human Planet 2016* arrives at the unsatisfactory conclusion that 85 percent of the World's population is now urban (Pesaresi *et al*., 2016, 41). In contrast, the U.N. Population Division, for example, estimated this share to be 54 percent in 2015 and projected it to increase to 56 percent by 2020. (U.N. Population Division, 2018).

A definition of the *Urban Footprint* based on the contiguity of built-up *Landsat* pixels, such as the one we adopted, is particularly useful for measuring changes in cities over time. This analysis was first conducted in the *Atlas of Urban Expansion—2012 Edition*, (Angel *et al*., 2012) for two time periods. It was expanded to three time periods and to a global sample of 200 cities in 2016, with the *Atlas of Urban Expansion—2016 Edition* (Figure A12). A more elaborate presentation of the methodology used in creating urban footprints is given in this edition of the *Atlas* (Angel *et al.,* 2016, 28-31).

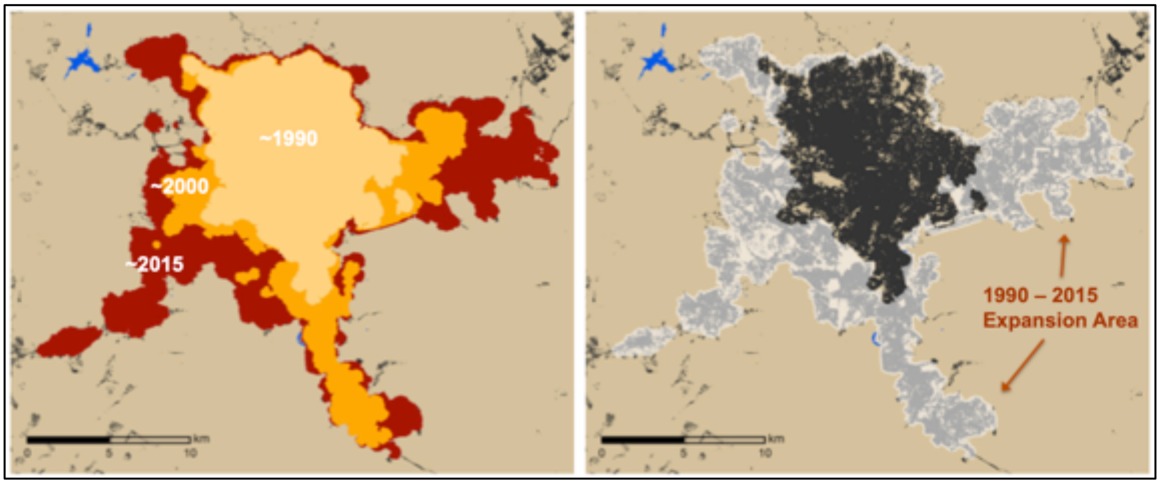


Figure A12: The urban footprint of Addis Ababa, measured in three periods using LANDSAT imagery (left). For some analyses, the area that developed between 1990 and 2015 was combined to form the Expansion Area (right).

6. Measuring City Population

The methodology for measuring the *Urban Footprint* that is described in the preceding section is universally replicable – it can be used in any city for which *Landsat* imagery is available, which is to say any city on earth. This addresses a key consideration when calculating metrics for global comparison, which is the comparability of the area of analysis. This is also described in some detail in the preceding section. However, the use of the urban footprint as an area of analysis also brings serious complications in terms of the aggregation of data. Because the urban footprint is defined in terms of the built-up area of the city, it rarely corresponds to the administrative and census boundaries that are used to gather social, economic and demographic information about that city. The methodology used for defining the study area (described in Step 1 of the urban footprint generation methodology) uses the administrative or census boundaries as the core aggregate units for forming the study area. This is done to enable the disaggregation and aggregation of key statistics from the level of the administrative area to the level of the urban extent. The first such measure that must be disaggregated and aggregated is *City population.*

City population is a necessary input for the calculation of the factors of density. Indeed, population of the urban footprint is the numerator in the standard *Urban Density* equation used in this paper (the denominator is *area of the urban footprint*). A common challenge in comparing densities among different cities is ensuring that the numerator and denominator are generated in comparable ways. We estimated the population living within the urban footprint using digitized maps for census enumeration districts (provided by CIESIN at Colombia University), as well as estimates provided by the Chinese Academy of Sciences for Chinese cities, and, in some cases, census enumeration maps directly procured from national census bureaus. The urban footprint overlaps the census enumeration districts, but may not include the entire area of a particular district. We adopted an apportionment strategy that relied on *Landsat* imagery of the urban footprint showing built-up and non built-up pixels. Each census enumeration district contains built-up pixels and non-built up pixels. Under this apportionment strategy, the population within each enumeration district is assumed to be living within the built-up pixels. It is further assumed that the population is equally divided among those built-up pixels. Each built-up pixel is assigned a proportionate amount of the total population of the census enumeration district. The pixels falling within the urban footprint are identified. The population of those pixels is summed, and that sum is added to the total population of the city. The sum of the populations of all of the built-up pixels within the urban footprint is the total population of the urban footprint, or the *City Population*.

7. Measuring *Residential Share*

The *Urban Footprint* of a city includes all of its residential areas and the urbanized open spaces in and around them that are clearly associated with them. It also includes, of course, other land uses: commercial, industrial, public, civic, transport, utilities, and public as well as private open spaces being the major ones. The measurement of *Residential Share* seeks to estimate the share of the urban footprint in residential use. Once again, the Halton sequence allows us to introduce a random sampling method to our gathering of this data. The *Halton Sequence* is used to select sample points within the urban footprint. Each point is checked to determine if it falls on a site that is in residential use. A sequence of Halton points is projected on a high-resolution *Bing* imagery of the urban footprint of a city. The *Bing* imagery provides visual information that an image analyst then uses to determine the underlying land use of the site on which the point falls (Figure A13).

Figure A13: A Halton point falling in a residential area in Bangkok, Thailand.

The points were sampled sequentially. 300 points were initially checked in each city and additional points were added based on the area of the city's urban footprint using a provisional formula. In practice, we sampled many more points than the numbers suggested by our formula and we sampled many more than were necessary had we adopted an appropriate stopping rule, but we did not have an appropriate stopping rule in mind at the time. Now that we can observe how fast the values for *Residential Share* converge (see figure A14), we can formulate a stopping rule.

For each sample point, the analyst determined if the use of the land on which the Halton sample point fell is 'residential' or 'non-residential'. Because this metric is concerned with the underlying use of the land, residential use means "of or relating to dwellings," and could include residential structures, garden sheds, mixed use buildings, lawns and gardens, open spaces around residences, parking facilities associated with housing, and any other structures or land uses occurring on a parcel that is determined to be residential.

The determination was not based on zoned land uses, but instead was done based on the morphology of the block. This relied on pattern recognition, based on the size and shape of the buildings occupying the block in question. Residential building form is distinct because most residences are designed in such a way that light can penetrate into almost every room. It is rare to have large inner spaces away from windows or airshafts, with no natural light reaching them, such as those found in office, commercial or industrial buildings. If there are buildings on the block that appear to be residential in nature, then the block is considered residential. This includes mixed-use blocks, provided they have residences on them. When a block is divided between residential and non-residential uses, the determination of the point is based on whether it lands on the residential or the non-residential side. Other factors aside from building shape are also taken into account, including the relationship of structures to each other and their degree of homogeneity.

The taxonomy of land uses that is utilized for distinguishing residential blocks from non-residential blocks was developed for the *Atlas of Urban Expansion: 2016 Edition*. It has since been tested through several more recent research projects. It is systematic and relies solely on observable features. Sample points may fall in locations like those shown in the images in figure A14. The middle image (marked as 0) has the point falling into areas of non-residential use and the left and right images (marked as 1) have points that have fallen into areas of residential use.

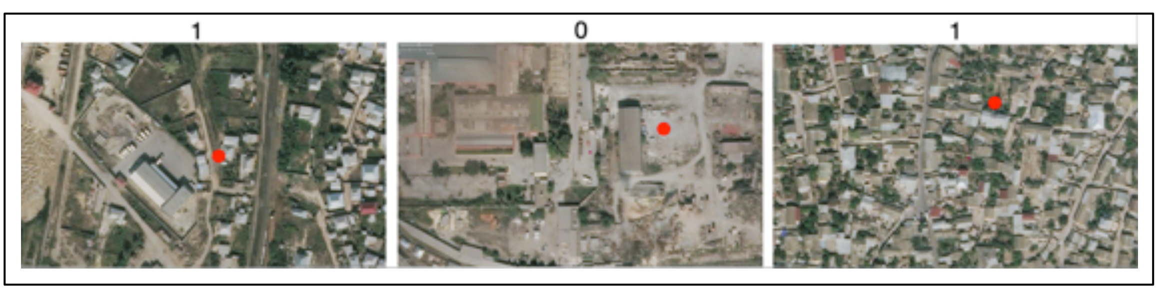


Figure A14: Assessment of land use, with Halton points shown in red.

Each of the sample points in a city is assessed in this manner. Once the pre-specified number of points has been gathered and their uses identified, *Residential Share* is calculated as a ratio of sums:

(A2) *Residential Share = Count of residential points ÷ Total number of sampled points*.

Although no stopping rules were used in deciding on the number of Halton points to be examined, we simulated the process outlined earlier after the fact:

(1) We looked at the first point in the sample. If it was in a residential area we added 1 to the 'residential score'. We also kept a 'point score' of 1.

(2) We calculated the first estimate of *Residential Share* as the ratio of the first 'residential score' and 'point score'. If the first point fell on a residential area, that ratio would be 1; otherwise it would be 0.

(3) We looked at the second point in the sample and added it to the cumulative 'point score'. If it was in a residential area, we added it to the cumulative 'residential score'.

(4) We calculated the second estimate of the cumulative *Residential Share* as the ratio of the cumulative 'residential score' and the cumulative 'point score'.

(5) We then added all the points for which we had data, one by one, keeping track of the cumulative 'residential score', the cumulative 'point score', and the cumulative *Residential Share*.

(6) Eventually, as expected, the cumulative *Residential Share* converged to a stable value.

(7) As we added more and more points, we also kept information on the cumulative *Residential Share* for the last 20 points added: Their average, standard deviation, their 95% confidence interval, and the ratio between the 95% confidence interval and their average value.

Since the 'last 20 points' stopping rule was employed after data collection was complete, we did not stop the data collection in time. In figure A15, we show the progress in *Residential Share* values as observations were added for the ten pilot cities. We clearly see that as more points were added, the values stabilized, and for each city they stabilized at a different value: In Kinshasa, for example, they stabilize at 45.6%, while in Wuhan they stabilize at 13.6%.

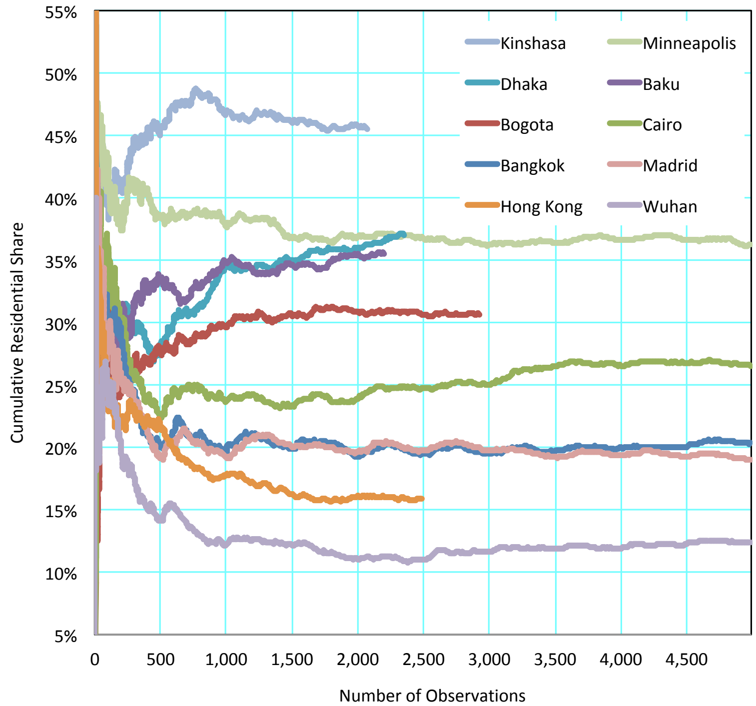


Figure A15: The values of the cumulative *Residential Share* in the ten pilot cities converged as more observations were added.

Table A4 displays the number of observations for each city and shows the results for the last 20 observations for that city, as well as the final estimated value for the city's *Residential Share*. The reader can ascertain that there is very little variation in these last observations: The 95% confidence intervals are all within 0.2% of the average values for these observations. This suggests that our estimates for the *Residential Share* in these ten pilot cities are indeed robust.

Residential Share	Kinshasa	Minneapolis	Dhaka	Baku	Bogota	Cairo	Bangkok	Madrid	Hong Kong	Wuhan
Observations	2,072	4,999	2,354	2,202	2,931	5,435	9,496	6,243	3,970	11,697
Average last 20 values	45.62%	36.16%	37.09%	35.54%	30.67%	25.93%	20.31%	18.78%	15.83%	13.55%
Standard Deviation	0.06%	0.04%	0.04%	0.05%	0.03%	0.02%	0.01%	0.01%	0.05%	0.00%
95% Confidence Level	0.03%	0.02%	0.02%	0.03%	0.01%	0.01%	0.00%	0.00%	0.02%	0.00%
Confidence Level (%)	0.07%	0.06%	0.05%	0.07%	0.05%	0.03%	0.01%	0.02%	0.14%	0.01%
Final Value	45.6%	36.1%	37.0%	35.5%	30.6%	25.9%	20.3%	18.8%	15.9%	13.6%

Table A4: The number of observations for calculating *Residential Share* in the ten pilot cities and data on the last 20 observations of the cumulative *Residential Share*, as well as its final estimated value.

8. Measuring *Plot Coverage*

Our next objective was to estimate *Plot Coverage*, defined as the average percentage of the land area of residential plots in a city that are covered by residential building footprints. Residential plots typically include a portion of the site that is occupied by buildings, and a portion of the site that is in open space, courtyard, or surface parking. *Plot Coverage* seeks to measure the total area of a sample of residential blocks (or the portions of those blocks in residential uses), and the total surface area of the residential building footprints within those blocks (see figure A16). The average *Plot Coverage* in the city is estimated as the ratio of the total area of residential buildings footprints and the total area of residential blocks.

A subset of the same Halton points that were used to measure *Residential Share* were used to measure *Plot Coverage*, though only points that had been tagged as residential selected for the *Plot Coverage* analysis. In addition, many fewer points were used due to the added complexity of digitizing block boundaries and footprint boundaries.

As before, the points were projected against current high-resolution *Bing* imagery of the urban footprint. The *Bing* imagery provided visual information that the analyst used to first digitize the boundary of the block area in residential use (see Figure A17); and then locate all the residential structures within this area and digitize their footprints, as defined by their rooflines (see Figure A16). The boundaries of contiguous residential areas were typically city streets that define residential blocks, but occasionally boundaries were found to be within blocks, when a change of use—to open space or to industrial, commercial, office, or civic uses—was detected.

The number of residential blocks or parts of blocks that were sampled in each city was supposed to be based on the area of its urban footprint, using a provisional formula. In practice, the nature of the pilot test led to many more locations that those planned for by the formula being analyzed. The numbers varied from as few as 196 residential blocks in Hong Kong, to 564 in Cairo, and 1,201 in Bangkok. The number of observations for each city is reported on in Table A4.

Residential blocks were examined one by one, their boundaries were digitized, and their areas were calculated. Next, all the residential building footprints within each block were digitized one by one, and the area of each one, as well as the total area of residential building footprints within the block, was calculated.

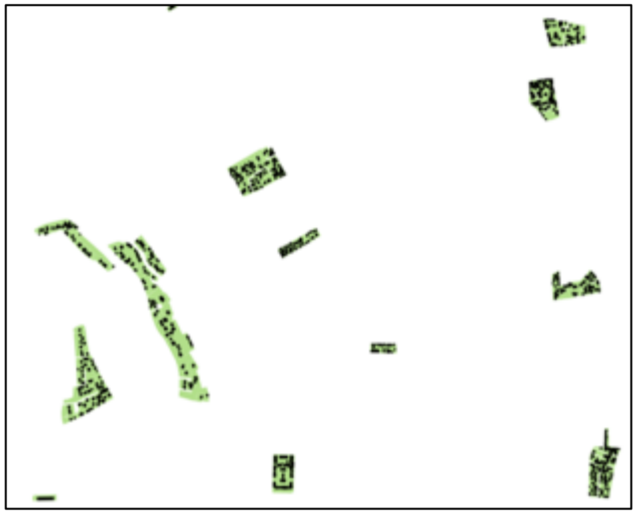


Figure A16: Residential blocks selected using Halton points and the residential building footprints within these blocks in a segment of the urban footprint of Bangkok, Thailand.

Capturing the footprint of the buildings means tracing the exterior walls of all residential buildings at ground level. Unfortunately, satellite imagery does not permit the analyst to view the backside of a building, so the roof shape was assumed to correspond to the building footprint. In this way, the footprints of all of the residential buildings in a residential block were digitized (Figure A18).

Figure A17: Outline of a residential block in Baku, Azerbaijan.

Figure A18: Outline of residential structures within a defined residential block in Baku, Azerbaijan.

The estimated average *Plot Coverage* in a city—given the sampled residential blocks and the building footprints within them—was then estimated as a ratio of two sums: (1) The sum of all the areas of all the residential building footprints within the sampled residential blocks; and (2) the sum of the areas of all the sampled residential blocks:

(A3) *Plot Coverage = Total area of Building Footprints ÷ Total area of residential blocks.*

As in the case of *Residential Share* reported on earlier, no stopping rules were used in deciding on the number of Halton points to be examined, and we only simulated the process of applying a stopping rule in the case of *Plot Coverage* after the fact. In estimating *Plot Coverage* for the ten pilot cities, we followed the following procedure:

(1) We looked at the first point in the sample that was classified 'residential', we identified the bounded 'residential area' around that point, and we digitized its boundaries and calculated its area.

(2) We then identified all the residential buildings within this area, digitized their footprints, and calculated their total 'building footprint area'.

(3) We then calculated the first estimate of *Plot Coverage* as the ratio of the first pair of 'building footprint area' and 'residential area'.

(4) We looked at the second 'residential' point in the sample, calculated the 'building footprint area' and the 'residential area' associated with it, and added its 'building footprint area' to the cumulative 'building footprint area', and its 'residential area' to the cumulative 'residential area'.

(5) We calculated the second estimate of the cumulative *Plot Coverage* in the city as the ratio of the cumulative 'building footprint area' and the cumulative 'residential area'.

(6) We then added all the points for which we had data, one by one, keeping track of the cumulative 'building footprint areas', the cumulative 'residential area', and the cumulative *Plot Coverage*.

(7) Eventually, as expected, the cumulative *Plot Coverage* for each of the ten pilot cities converged to a stable value.

(8) As we added more and more points, we also kept information on the cumulative *Plot Coverage* for a stopping rule based on the last 20 points added: Their average, standard deviation, their 95% confidence interval, and the ratio between the 95% confidence interval and their average value.

Again, since the 'last 20 points' stopping rule was employed after data collection was complete, we did not stop the data collection in time. In Figure A19, we show the progress in *Plot Coverage* values as observations were added for the ten pilot cities. We clearly see that as more points were added, the values stabilized, and for each city they stabilized at a different value: In Dhaka, for example, they stabilized at 52.5%, in Madrid at 26.0%, and in Minneapolis at 11.2%.

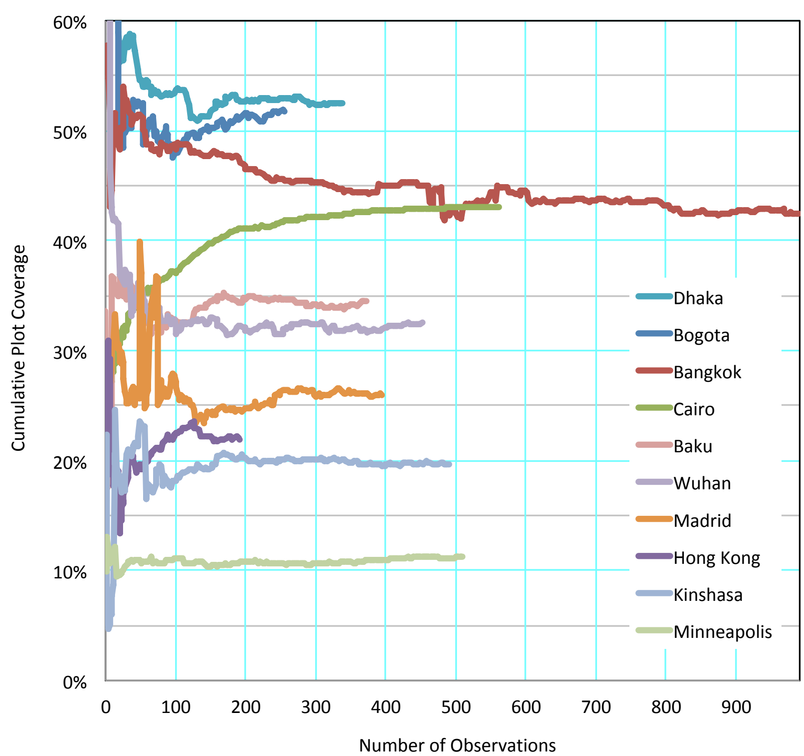


Figure A19: The values of the cumulative *Plot Coverage* in the ten pilot cities converged as more observations were added.

Table A5 displays the number of observations for the cumulative *Plot Coverage* in each city and shows the results for the last 20 observations for that city, as well as the final estimasted value for *Plot Coverage*. The reader can ascertain that there is very little variation in these last observations: The 95% confidence intervals are all within 0.25% of the average values for these observations. This suggests that our estimates for the *Plot Coverage* in these ten pilot cities are indeed robust.

Plot Coverage	Dhaka	Bogota	Bangkok	Cairo	Baku	Wuhan	Madrid	Kinshasa	Hong Kong	Minneapolis
Observations	339	257	1,021	564	374	453	396	490	196	511
Average last 20 values	52.51%	51.76%	42.65%	43.05%	34.33%	32.47%	25.94%	19.71%	22.10%	11.20%
Standard Deviation	0.05%	0.13%	0.03%	0.00%	0.18%	0.01%	0.08%	0.07%	0.09%	0.02%
95% Confidence Level	0.02%	0.06%	0.01%	0.00%	0.08%	0.01%	0.04%	0.03%	0.04%	0.01%
Confidence Level (%)	0.04%	0.11%	0.03%	0.00%	0.25%	0.02%	0.14%	0.16%	0.19%	0.07%
Final Value	52.5%	51.9%	42.7%	43.1%	34.5%	32.5%	26.0%	19.7%	22.1%	11.2%

Table A5: The number of observations for calculating *Plot Coverage* in the ten pilot cities and data on the last 20 observations of the cumulative *Plot Coverage* and its final estimated value.

9. Measuring *Building Height*

The estimation of the average *Building Height* in a city required information on the height of a sample of residential buildings, where height was measured as the total number of residential floors. In mixed use buildings, parking floors and commercial floors or other non-residential floors were excluded from the floor count whenever they could be detected. In these cases of exclusion it was assumed that the non-residential use occupied the entire floor. Techniques to measure building height were developed at the Urban Expansion Observatory and gvSIG Association, with the support of New York University.

The same set of Halton points used for sampling *Residential Share* and *Plot Coverage* were used to sample residential buildings for calculating *Building Height*. The points were projected against current high-resolution Bing imagery of the urban footprint. The Bing imagery provided visual information that the analyst could use to count the number of floors in those structures. The Halton points were sampled sequentially, and each sampled point corresponded to a residential building that was digitized and had its height measured. Again, we constructed a formula for estimating how many buildings were to be investigated in each city, based on its urban footprint. At a minimum, we sampled 914 buildings in Dhaka, double that number in Bangkok, Wuhan, and Madrid, and as many as 3,000 in Minneapolis (see Table A6).

Given a Halton point, the analyst first determined whether or not the point landed in a residential area. If a point was found to have landed in a residential areas, the analyst then identified the nearest residential building in that parcel. This was done by measuring the distance to the walls of all of the nearest residential buildings and choosing the one that was the closest to the point. The outer perimeter of this building's footprint was then digitized. This consisted of tracing the outline of the footprint of the building, based on what was shown in the Bing imagery. The footprint provided the area of the building at ground level (Figure A20). This procedure has since been updated to require the analyst to only digitize buildings on which a Halton point lands directly.

The analyst then counted the number of floors. In general, floors were counted by identifying window openings and balconies. We assumed that each window or balcony in a vertical row represented one floor. Where these features were not visible, comparisons could sometimes be made with adjacent buildings with visible windows. As a final measure, where adjacent buildings did not have visible windows but more distant buildings did, the shadows of buildings could be used to compare numbers of floors. In the majority of cases it was possible to measure the number of floors based on visible windows (Figure A21).

Figure A20: Digitized footprint of multi-story residential building in Baku, Azerbaijan.

Figure A21: Windows marked with red dots indicating the number of floors in an 8 story residential building in Baku, Azerbaijan.

The estimated average *Building Height* in a city—given all the sampled residential buildings—was then estimated as the mean of the building height values of each of the sampled points.

(A4) *Building Height = Total height of all sampled buildings ÷ Total number of sampled buildings.*

As in the case of *Residential Share* and *Plot Coverage* reported on earlier, no stopping rules were used in deciding on the number of Halton points to be examined to determine the average *Building Height* in a city, and we only simulated the process of applying a stopping rule in the case of *Building Height* after the fact. In estimating *Building Height* for the ten pilot cities, we followed the following procedure (detailed in a previous section):

(1) For each building in the city, measure the number of residential floors.

(2) Add the number of floors of the first building in the sample (reflecting its position as the first residential building to be digitized in the Halton sequence) to the number of floors in the second building in the sample to obtain a cumulative number of floors;

(3) Calculate the estimate of the *Building Height* in the city by dividing the cumulative number of floors by the number of sampled buildings;

(4) Continue to add values for more buildings to the cumulative building height, and divide by the number of sampled buildings to get a cumulative average building height for the city;

(5) The resulting *Building Height* will gradually converge to an estimated value for the city as a whole.

(6) Employ a stopping rule: For example, stop adding buildings when the 95% confidence interval for the last 20 cumulative *Building Height* estimates falls below an agreed-upon value, say below 0.5% of the average *Building Height* for these 20 observations.

Again, since the 'last 20 points' stopping rule was employed after data collection was complete, we did not stop the data collection in time. In Figure A22, we show the progress in *Building Height* values as observations were added for *Building Height* stabilized, and for each city they stabilized at a different value: In Hong Kong, for example, they stabilized at 23.0 floors, in Madrid at 4.7 floors, and in Kinshasa at 1.1 floors.

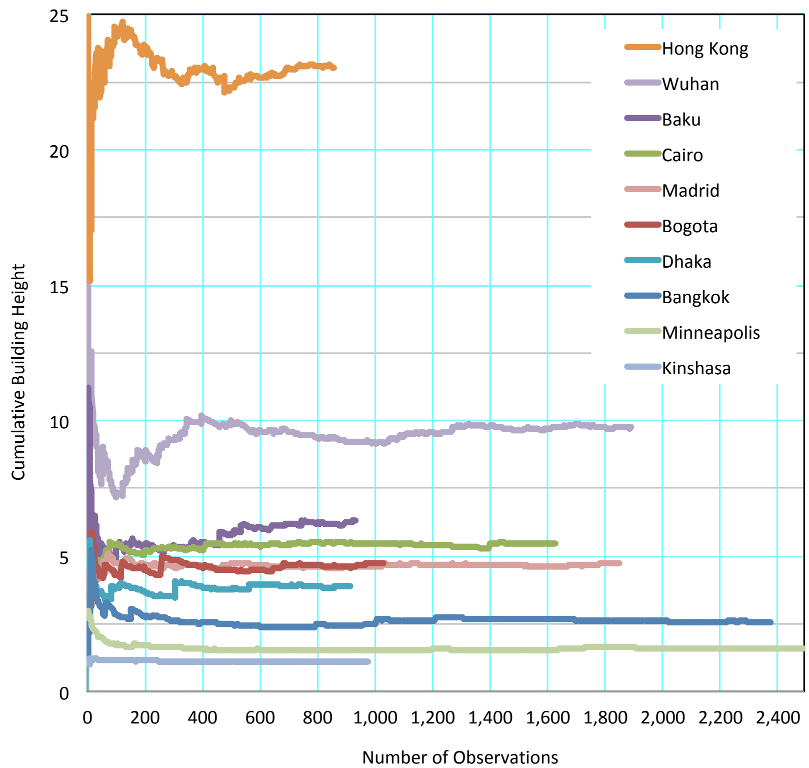


Figure A22: The values of the cumulative *Building Height* in the ten pilot cities converged as more observations were added.

Table A5 displays the number of observations for *Building Height* in each city and shows the results for the last 20 observations for that city and the final estimated value for its *Building Height*. The reader can ascertain that there is very little variation in these last observations: The 95% confidence intervals are all within 0.20% of the average values for these observations. This suggests that, if our individual estimates of the heights of sampled buildings were correct, then our cumulative estimates for the *Building Height* in these ten pilot cities are sensible.

Building Height	Hong Kong	Wuhan	Baku	Cairo	Bogota	Madrid	Dhaka	Bangkok	Minneapolis	Kinshasa
Observations	860	1,895	937	1,628	1,033	1,850	914	2,377	2,984	980
Average last 20 values	23.05	9.74	6.29	5.46	4.73	4.71	3.91	2.57	1.56	1.11
Standard Deviation	0.03	0.01	0.03	0.00	0.01	0.00	0.01	0.00	0.01	0.00
95% Confidence Level	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Confidence Level (%)	0.06%	0.04%	0.19%	0.03%	0.05%	0.05%	0.08%	0.04%	0.00%	0.02%
Final Value	23.0	9.7	6.3	5.4	4.7	4.7	3.9	2.6	1.6	1.1

Table A6: The number of observations for calculating *Building Height* in the ten pilot cities and data on the last 20 observations of the cumulative *Building Height*.

10. Measuring *Floor Plan Efficiency*

The measurement of *Floor Plan Efficiency* seeks to calculate the share of the gross floor area of a residential building that is taken up by private living areas. This means identifying all non-private areas such as stairwells, corridors, common areas and lobbies, parking floors, open floors, elevator shafts and mechanical spaces. The larger the share of the gross floor area devoted to those uses, the smaller will *Floor Plan Efficiency* be.

The measurement of this factor of density starts with the verifiable assumption that taller buildings have more of their typical floor areas occupied by common and mechanical spaces. The taller the building, the more fire exit stairwells, elevators, and mechanical equipment shafts it requires. Lower buildings, walk-up apartment buildings, for example, have less of their gross floor areas occupied by such uses. All of the gross floor area In single-family homes, except for the areas occupied by garages and walls, can be considered as living areas.

To estimate *Floor Plan Efficiency* in the ten pilot cities, we initially assumed that it is simply a function of *Building Height,* and that it does not otherwise vary from city to city. To render this assumption useful, we gathered data on the proportion of living space in buildings of various heights. Architectural floor plans were collected for dozens of different buildings. These plans were gathered via searches in the World Wide Web. Many of them came from real estate websites. Others came from the sites of architecture firms. Still others were included in dissertations on vernacular building styles, or in industry white papers. Only plans that clearly distinguished between living areas and non-living areas were included. In total, roughly fifty plans were collected.

Plans were collected for buildings of varying sizes and heights and grouped into three 'building types' for the purpose of analyzing *Floor Plan Efficiency*: (1) Single-family; (2) Non-core multi-family; and (3) Core multi-family (Figure A23).

Figure A23: Single-family attached homes in Bogotá, Colombia; Non-core multi-family housing in Brooklyn, New York; and Core multi-family housing in Singapore.

Single-family dwellings can be detached, attached to a second unit, or arranged in rows with common walls to form townhouses. They must have their own private entrance with no intervening interior common areas. 'Core' refers to buildings with a central mechanical shaft that contains elevators, lobbies, and other equipment. Core multi-family buildings are tall (usually greater than eight stories) and typically house a large number of residents. Non-core multi-family buildings include walkup apartments and low-rise and mid-rise buildings without cores.

For this part of our analysis, only enclosed areas of residential buildings were considered as 'floor areas'. Areas that were exclusively within a private dwelling unit were categorized as 'living areas.' Common areas (lobbies, staircases, elevators, etc.), shared facilities (resident's lounge, children's playroom, etc.), and enclosed inaccessible areas (ducts and shafts) were categorized as 'non-living areas.' Garages and storage areas, rooftop gardens, balconies, and verandas were not included as parts of the floor areas of residential buildings.

Image analysts digitized the architectural plans by importing them as PDF or JPG files into QGIS and using the available GIS tools to outline the internal and external partitions shown in the plan. They classified each interior space as either a 'living area' or a 'non-living area.' When the digitization was complete, it was possible to calculate the average share of 'floor area' in a given plan that constituted 'living area' for each building type. *Floor Plan Efficiency* for a floor plan for an individual building was calculated as a ratio:

(A6) *Floor Plan Efficiency = Living area ÷ Floor area.*

For each building type, the average *Floor Plan Efficiency* is calculated as a ratio of sums:

(A7) *Floor Plan Efficiency = Total Living area in all plans ÷ Total floor area in all plans.*

When these calculations were complete, the analysts revisited all of the buildings whose footprints were digitized in the earlier measurement of *Building Height*. Each of these buildings was identified and tagged into one of the three 'building types' – 'single family,' 'non-core multi-family,' or 'core multi-family.' Based on this classification, the average *Floor Plan Efficiency* for the city as a whole could be calculated. This correction allowed us to estimate the total estimated 'living area' in the ten pilot cities.

In some cases more idiosyncratic methods were used. For instance, cities like Baku, Azerbaijan, while part of the former Soviet Union, used prefabricated construction methods that employed thicker walls. A more precise methodology was available in Minneapolis, Minnesota, where detailed information on individual buildings was available at the level of the municipality. The values calculated using that information were extrapolated to the urban footprint as a whole. Using that as a benchmark, we made the assumption that outer walls in Minneapolis, Minnesota, would be thicker than outer walls in Kinshasa, due to the employment of insulation, and reduced the Kinshasa estimate accordingly. The value for Wuhan was based on a rule of thumb used by real estate brokers there. The research team applied corrections to the *Floor Area Efficiency* estimates for individual cities based on this information. These estimates should therefore be considered preliminary and subject to future revision.

11. Measuring the *Occupancy Rate*

The *Occupancy Rate* of residential units was estimated from publicly available data. The estimation of this factor required data on the share of all residential dwelling units that are currently occupied. While most cities have occupancy rates of 90% or higher, some cities have large numbers of dwelling units that sit empty—either because they are new and still unsold, because they have come into the market for resale, because they are kept empty for occasional use by their out-of-town owners, because they are kept empty for fear of being unable to evict tenants, or because they were abandoned for one reason or another.

UN-Habitat developed several methodologies for measuring vacancy rates in cities, documented in a paper titled "Ghost Cities and Empty Houses: Wasted Prosperity" (Lòpez Moreno and González Blanco, 2014). Gonzales Blanco, one of the co-authors of that early paper was engaged as a member of our research team to gather comparable data for the global sample of 200 cities*.* The methodology used in estimating the *Occupancy Rate* is described in detail in *Guidelines for Estimating the Housing Occupancy Rate in Urban Contexts* (Gonzáles Blanco, 2019) and will only be summarized here.

Assuming that there is no difference in the occupancy rate of dwelling units of different sizes, the *Occupancy Rate* for a city is calculated as:

(A8) *Occupancy Rate = Total Occupied Dwelling Units ÷ Total Number of Dwelling Units = Total Occupied Living Area ÷ Total Living Area.*

In only three of the ten pilot cities—Cairo, Madrid, and Minneapolis—was it possible to estimate the *Occupancy Rate* within their urban footprint with national census data. When census data on the number of occupied housing units was not available, but data on the total number of housing units was available, the number of occupied housing units was estimated by subtracting the census category 'households sharing the same housing unit' from the census category 'total number of domestic households.' Assuming that every household occupied a single dwelling unit, this number yielded an estimate of the number of occupied units in the city. The occupancy rates in Bangkok, Bogotá, Dhaka, and Hong Kong were estimated using this method.

A third approach for estimating the *Occupancy Rate* was employed when neither the 'occupied housing units' nor 'total housing units' have been reported on in any database accessible to the public, but there was data available on the average household size or on the number of households in the city, as well as on the total residential floor area in the city and the average floor area per person there. Again, the number of 'occupied dwelling units' was estimated by subtracting the census category 'households sharing the same housing unit' from the census category 'total number of domestic households.' The total number of dwelling units was calculated by dividing the total square meters of residential floor space in the city by the average dwelling unit size, estimated by multiplying floor area per person and average household size. This methodology was employed in Baku, Kinshasa and Wuhan. We note that this third approach is subject to large errors, and that the estimate of the *Occupancy Rate* in Baku, Kinshasa and Wuhan is tentative.

12. Measuring *Persons per Dwelling Unit*

The final metric required in order to calculate the factors of *Urban Density* is *Persons per Dwelling Unit.* This metric is rarely available from the census directly, but can easily be calculated for the administrative units (typically census enumeration districts, but sometimes municipal boundaries) that intersect the urban footprint of a given city, as long the total population and the total number of dwelling units are known. In some countries this second value was only available inferentially, as described in the preceding section on *Occupancy Rate*. In Azerbaijan, for example, the census provided the total residential floorspace in the municipal boundary (which does not correspond to the urban footprint), average square meters per person, and average household size, from which the total number of dwelling units was estimated. *Persons per dwelling unit* within an administrative unit is calculated as:

(A9) *Persons per Dwelling Unit = Total Population ÷ Total Dwelling Units*

In some cases, this value will be the same as the value given in the census for household size, indicating that the census does not allow more than one household to be counted in a dwelling unit. In other cases the numbers will differ, indicating that the census allows respondents to have more than one household within a dwelling unit.

The simple calculation in A9 provides a value for each of the administrative units that intersect the urban footprint. Much as in the calculation of *City Population*, a strategy is needed to aggregate these values into one mean value for the *Urban Footprint*. Because of the essentially random variation in the size and population of administrative units, this is best calculated as a weighted mean, on the basis of the population of each administrative unit falling within the urban extent. This is shown in Table A7.

Administrative Unit	*(a) Total Population*	*(b) Population within the urban extent*	*(c) No. of dwelling units*	*(c) Persons per dwelling unit ($\frac{a}{c}$)*	*(d) Share of urban extent population*	*Weighted Persons per dwelling unit (d×c)*
A	200,000	85,000	62,500	3.2	17%	0.544
B	100,000	45,000	20,000	5	9%	0.45
C	450,000	425,000	125,000	3.6	74%	2.664
Total	750,000	555,000	189,500	3.96		3.658

Table A7: The value *Persons per dwelling unit* for the urban footprint is computed as the population weighted mean of the values for the administrative units that intersect the urban footprint.

This population-weighted mean value of *Persons per dwelling unit* is applied to the *Urban footprint* and is later used to estimate the total number of dwelling units in the city.

13. Estimating the Overall Measurement Error in Our Calculated Values for *Dwelling Unit Size*

The methodology for collecting city data for the eight metrics described above allowed us to calculate *Urban Density* and all its factors in the ten representative cities with simple arithmetical operations of multiplication and division. As expected, there are errors in our data. In three of the eight metrics—*Residential Share*, *Plot Coverage* and *Building Height*—the errors, as shown above, are of the order of ±0.25%. The overall accuracy of our classifications of *Landsat* imagery for determining the *Urban Footprint* is of the order of 83% (Blei *et al*., 2018), so the error in estimating the area of the *Urban Footprint* in the ten cities studied may be of the order of ±17%. It is difficult to assess the possible errors in estimating the *Urban Population* within the *Urban Footprint*, but it may also be of the order of ±10%. The data for *Persons per Dwelling Unit* was obtained directly from census documents pertaining to an area that was slightly different from our *Urban Extent*. We cannot assess the error there, but it may be of the order of 5%. Finally, our estimates of *Floor Plan Efficiency* were ballpark estimates based on a small sample of floor plans that may entail large errors, possibly of the order of ±20%.

Alonso (1968) explains that for models involving multiplication (or division), the error for a product (or the ratio) is calculated as follows:

(1) $z = xy$ or $z = x/y$.

(2) $e_z^2 = y^2e_x^2 + x^2e_y^2$, $e_z^2 = e_x^2/y^2 + e_y^2/x^2$ if x and y are not correlated. If they are correlated, then

(3) $e_z^2 = y^2e_x^2 + x^2e_y^2 + xye_xe_yr$, where r is the correlation coefficient.

Since many of the values in our models were derived from other values, they are subject to cumulative errors. This is especially true of our estimates of the total residential floor area in the city, the total occupied floor area, dwelling unit size, or occupied floor area per person. In this Technical Annex we compare our derived values for *Dwelling Unit Size* with values found in published articles and reports. As we shall see below, in most of cities we studied, the values are within a reasonable range of each other.

The average *Dwelling Unit Size* in each of the ten cities studied was obtained arithmetically from seven out of the eight metrics for which we collected data.

We observe that one of the metrics we collected data for, *People per Dwelling Unit*, is a product of two of the seven factors that together make up *Urban Density—Dwelling Unit Occupancy* and the *Occupancy Rate*:

(4) *Dwelling Unit Occupancy × Occupancy Rate = (Total Population ÷ Total Number of Occupied Dwelling Units) × (Total Number of Occupied Dwelling Units ÷ Total Number of Dwelling Units) = (Total Population ÷ Total Number of Dwelling Units = People per Dwelling Unit.*

We can therefore represent *Urban Density* as a product of six factors, replacing *Dwelling Unit Occupancy* and the *Occupancy Rate* with *Persons per Dwelling Unit:*

(5) *Urban Density = Persons per Dwelling Unit × Dwelling Unit Packing × Floor Plan Efficiency × Building Height × Plot Coverage × Residential Share.*

We also know that *Urban Density* is a ratio of two metrics and that *Dwelling Unit Size* is the reciprocal of *Dwelling Unit Packing*:

(6) *Urban Density = Total Population ÷ Urban Footprint.*

(7) *Dwelling Unit Size = 1 ÷ Dwelling Unit Packing.*

These equations allow us to represent *Dwelling Unit Size* in terms of seven of the eight metrics for which we collected data.

(8) *Dwelling Unit Size =* (*Persons per Dwelling Unit × Floor Plan Efficiency × Building Height × Plot Coverage × Residential Share) ÷ Urban Density =* (*Persons per Dwelling Unit × Floor Plan Efficiency × Building Height × Plot Coverage × Residential Share × Urban Footprint) ÷ Total Population.*

Equation (8) allows us to calculate *Dwelling Unit Size* (and, of course, its reciprocal *Dwelling Unit Packing*) from data for seven metrics, and this is how we obtained its value. As expected, because this value was obtained from not less than six multiplications and divisions, its cumulative error can be expected to be quite large. To assess the magnitude of this error, we compared our calculated estimates for *Dwelling Unit Size* with values in published reports and articles. Table A3 below provides the data for the seven metrics for each of the ten cities, our estimates of *Dwelling Unit Size* from these data, and the values for

Dwelling Unit Size found in the literature. Figure A5 below compares our estimate of *Dwelling Unit Size* with values found in the literature.

Metrics Obtained from Primary & Secondary Data		Dhaka	Hong Kong	Kinshasa	Bogotá	Cairo	Baku	Madrid	Bangkok	Wuhan	Minnea-polis
a.	Population ('000)	13,609	4,322	10,226	7,802	15,735	1,672	5,256	14,011	8,174	2,627
b.	Urban Footprint (hectares)	36,541	12,278	45,681	39,723	136,396	25,662	84,407	294,462	183,723	251,256
c.	Building Height (stories)	2.5	20.5	1.1	2.8	4.4	2.6	3.4	1.9	5.8	1.4
d.	Plot Coverage	53%	22%	20%	52%	43%	35%	26%	44%	32%	11%
e.	Residential Share	37%	16%	46%	31%	26%	35%	19%	20%	14%	36%
f.	Persons per Dwelling Unit	4.2	2.8	5.1	3.6	2.1	3.8	2.3	3.0	2.2	2.4
h.	Floorplan Efficiency	85%	75%	95%	87%	79%	67%	83%	89%	75%	90%
Calculated from the above Metrics											
t.	Dwelling Unit Packing (persons/occupied D	212	235	472	140	142	81	201	105	104	87
v	Dwelling Unit Size (sq.m.) [Reciprocal of t]	47	43	21	71	70	123	50	95	96	115
Values Reported in the Literature											
x.	Dwelling Unit Size (sq.m.)	73	45	32	86	80	59	72	115	92	167

Table A3: Data for the seven metrics used to calculate *Dwelling Unit Size* in the ten cities studied and a comparison of our estimate of *Dwelling Unit Size* with values found in the literature.

For five of the ten cities studied, our estimates of *Dwelling Unit Size* are very close to those obtained from outside sources: Hong Kong (CommSec, 2017), Bogota (CAMACOL, 2018 and DANE, 2019), Cairo (USAID, 2008), Bangkok (Thailand-Property.com, 2019), and Wuhan (CEIC, 2017). The values we estimated for Dhaka (Nancy, 2016), Kinshasa (Halleux, 2019), Minneapolis (U.S Census Bureau, 2013) and Madrid (Arnaz, 2015) are 35%, 34%, 31%, and 31% lower than those reported on by outside sources, discrepancies that could well be within the cumulative error range. The only discrepancy that stands out is the estimated value for Baku, which is 208% of the value reported in the literature (World Bank, 2010). This is worrisome, because there is no question that the value we estimated is too high. Yet after considerable efforts to unearth possible errors in our calculations we have failed to uncover the source of this discrepancy and it is therefore left for further investigation at a later date. Still, with the possible exception of Baku we believe that these estimates indicate that the measurement strategy described in this Annex is generally robust enough to be used for calculating the factors of *Urban Density* in other cities, and that—with some refinement—these methods can be used to track changes over time in these factors in single cities as well.

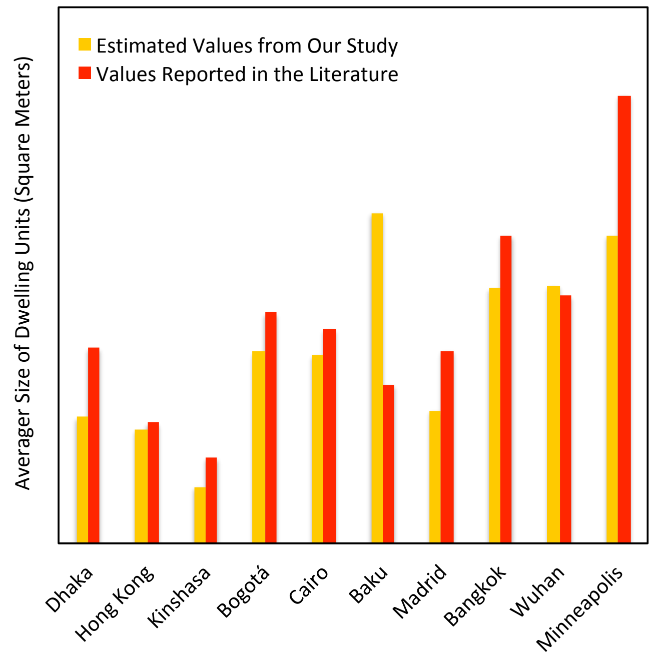


Figure A5: A comparison of Dwelling Unit Size for the ten pilot cities calculated as a ratio of Population and the product of six remaining metrics (red) with estimates from outside sources (orange).

Conclusion

This Methodological Annex described a set of procedures for obtaining reliable estimates of the eight primary metrics that are used to calculate the factors of *Urban Density*. We believe that this is a significant development with important policy implications because, as we noted earlier, addressing the factors that constitute *Urban Density* one by one with a view to registering its contribution to progress in densification requires that we measure them properly.

The proposed methodology is relatively simple and robust, relying in the main on globally available satellite imagery and on published information from national censuses. Moreover, the methodology could be used to generate comparable data for all cities, countries, and regions, so as to better monitor the global contribution of urban densification to the mitigation of climate change.

In obtaining values for the eight primary metrics that are used to calculate the factors of *Urban Density* for the ten pilot cities, we have sampled too many points. In the future, the stopping rules proposed and discussed here can be used to considerably reduce the number of points to be tested. In addition, it may be possible to obtain reliable values for *Plot Coverage,* for example, without resorting to the digitization of the boundaries of residential blocks and building footprints, by dropping large numbers of Halton points into the urban footprint and estimating the share that fall on residential roofs as well as the share of points that fall on residential open space. *Plot coverage* can then be calculated as the cumulative ratio of the number of points falling on roofs and the total number of points falling on residential areas, both those falling on roofs and those falling on residential open space. Other simplifications may be in order as well, as we proceed to estimate these factors in all 200 cities in the global sample of cities.

This work is now in its initial stages and the results reported here must be interpreted with caution. Some of the results presented here, especially those for Baku, are currently being re-examined. Still, we believe that even at his stage one can see the potential of decomposing *Urban Density* into measurable factors as a step forward on the way to densifying our cities in a pragmatic and systematic manner in the years to come.

* * *

Technical Appendix References

Alonso, W., 1968. Predicting Best with Imperfect Data, *Journal of the American Institute of Planners*, 34(4), 248-255.

Angel, S., 2012. *Planet of Cities*. Cambridge, MA: Lincoln Institute of Land Policy.

Angel, S, Blei, A. M., Lamson-Hall, P., Galarza Sanchez, N., Parent, J, and Civco, D.L., 2016. *Atlas of Urban Expansion—2016 Edition*. Cambridge, MA: Lincoln Institute of Land Policy.

Angel, S., Lamson, P., Hall, B. G., Liu, Y., Galarza, N., & Blei, A. M. (2018). Our Not‐So‐Urban World (No. 42). Marron Institute of Urban Management Working Paper.

Arnaz, R., 2015. ¿Casa grande y barata? Prueba en castilla-la mancha o Extremadura, pero huye de euskadi o Madrid. *Idealista News*.

CAMACOL 2018. Estudio de Oferta y Demanda de Vivienda y No Habitacionales. Cámara Colombiana de la Construcción. Bogota, Colombia.

CEIC, 2017. China Floor Area of Residential Building per Capita: Urban: Hubei: Wuhan. CEIC Data. New York.

CommSec, 2017. CommSec Home Size Trends Report. Commonwealth Securities Research, Commonwealth Bank of Australia. Canberra, Australia.

DANE, 2019. *Vivienda Vis y no Vis.* Departamento Administrativo Nacional de Estadística. Bogota, Colombia.

Gonzáles Blanco, Z., 2019. *Guidelines for Estimating the Housing Occupancy Rate in Urban Contexts*, New York: Marron Institute.

Halleux. J-M., 2019. Private email communication with author, 2 April.

López Moreno, E., and González Blanco, Z., 2014. "Ghost Cities and Empty Houses: Wasted Prosperity." *American International Journal of Social Science*, 3(2), pp 207-216.

Nancy, S. J. (2016). Densification of Residential Areas in Dhaka: An Enquiry of Causes and Consequences to Address Approaches towards Sustainability. Doctoral Thesis. Bangladesh University of Engineering and Technology. Dhaka, Bangladesh.

Pesaresi et al, eds., 2016. Atlas of the Human Planet—2016, Joint Research Centre, European Commission.

Thailand-Property.com, 2019. *Bangkok condo Market insight.* Thailand-Property.com,

The Math Forum, 2003. "Ask Dr. Math, Average of Ratios vs. Ratio of Averages." National Council of Teachers of Mathematics.

U.S. Census Bureau, 2013. 2013 Housing Profile: Minneapolis St. Paul, MN-WI. American Housing Survey Factsheets. United States Bureau of the Census. Washington, D.C.

U.N. Habitat, 2013. *Streets as Public Spaces and Drivers of Urban Prosperity*. Nairobi: United Nations Human Settlements Programme. http://unhabitat.org/books/streets-as-public-spaces-and-drivers-of-urban-prosperity/

U.N. Population Division, 2018. *World Urbanization Prospects—2018*, file 2.

USAID, 2008. *Housing Study for Urban Egypt*. United States Agency for International Development. Washington, D.C.

World Bank, 2010. *Azerbaijan - Living conditions assessment report (English)*. Washington, DC: World Bank.

Blei, A.M., Angel, S., Civco, D. L., Liu, Y., and Zhang, X., 2018. Accuracy Assessment and Map Comparisons for Monitoring Urban Expansion: The Atlas of Urban Expansion and the Global Human Settlement Layer, Working Paper WP18AB1, Cambridge MA: Lincoln Institute of Land Policy. Online at: https://www.lincolninst.edu/sites/default/files/pubfiles/blei_wp18ab1.pdf

Endnotes

1 In principle, other things being equal—i.e. disregarding variations in the overall shape of its footprint, in internal spatial variations in density, or in internal arrangements of land use—everything in a more compact city will be closer. Differences in the internal distribution of

2 In a recent review of evidence from 300 studies, Ahlfeldt and Pietrostefani showed that the measurable effects of compact urban form have tended to become more positive over time (2018, 19). More specifically, compact and higher density urban growth has been associated with reduced greenhouse gas emissions attributed to transport (National Research Council, 2009, quoted in Seto *et al*., 2014, 948). Norman, Maclean and Kennedy (2006) found that in Toronto, Canada, for example, low-density suburban development is 2.0-2.5 times more energy and greenhouse gas intensive on a per-capita basis than high-density urban core development. Densification can reduce greenhouse gas emissions by increasing the viability of walking and bicycling (Saalens et al, 2003; Naess, 2005, Zhou and Kockelman, 2008); by making public transit more viable (Bunting et al., 2002; Holtzclaw et al., 2002; Saelens et al., 2003; Forsyth et al., 2007); and by shortening average travel distances (Frank and Pivo, 1994; Cervero and Kockelman, 1997; Ewing and Cervero, 2001; Brownstone and Golob, 2009).

3 Bourne (2001) notes that a single focus on residential densities, as all but the last four metrics in table 1 do, may fail to account for the effect of low-density commercial and industrial uses on the overall compactness of cities, particularly where sustainability issues are concerned. Indeed, since population data is collected at people's homes, *Urban Density* as defined here is a night-time density, i.e. a residential density rather than an employment density. But as we shall show, overall *Urban Density* can increase if the *Residential Share* of land use in the city increases at the expense of commercial and industrial uses, i.e. when overall employment density increases. *Urban Density* thus includes employment density.

4 The same anatomy of density relationships pertains to all other spatial units introduced in table 1 as well.

5 We note at the outset that an 'average' ratio of two quantities can have two distinct meanings (The Math Forum, 2003). For example, the *Average Building Height* in a city, measured in floors, could be the ratio of the total *Floor Area* in the city and the total area of *Building Footprints* in the city. Alternatively, it could be the *average (or mean) height of individual buildings in the city*, which would be the sum of the heights of individual buildings (which are the ratios of their floor areas and building footprints) divided by the total number of buildings. In this presentation, all of the average values of the factors that constitute *Urban Density* are defined as the ratio of sums and not as mean values of individual observations.

6 A detailed definition of the urban footprint and the method of mapping and calculating urban footprints using *Landsat* satellite imagery is given in Angel et al., 2016, *Volume 1: Areas and Densities*, 21-30.

7 E.g. stairwells, corridors, and elevator shafts.

8 The reader can easily ascertain that the product of *Floorspace Occupancy* and *Floor Area Ratio* yields the familiar *Net Residential Density*, a common measure of the average number of people (unfortunately, the same term is also used for the number of dwelling units) in a hectare of net residential area in the city.

9 The reader may note that an important measure of residential overcrowding (see, e.g. Gove, Hughes and Gall, 1983; Blake, Kellerson and Simic, 2007; Dol and Haffner, 2010, table 2.1, 51;), *Occupied Floor Area per Person*, is simply the ratio of *Dwelling Unit Size* and *Dwelling Unit Occupancy*, but only if we assume that the size of occupied and unoccupied dwelling units is the same.

(1) *Occupied Floor Area per Person = Dwelling Unit Size ÷ Dwelling Unit Occupancy = (Total Living Area in Occupied Dwelling Units ÷ Total Number of Occupied Dwelling Units) ÷ (Total Population ÷ Total Number of Occupied Dwelling Units) = Total Living Area in Occupied Dwelling Units ÷ Total Population = Occupied Floor Area per Person.*

If, as we suspect, the average size of unoccupied dwelling units is larger than the average size of occupied dwelling units, then a correct estimate of *Occupied Floor Area per Person* may be smaller than that estimated here.

10 The reader should keep in mind that we can decompose *Urban Density* into factors in other ways as well, incorporating other well-known and less well-known density metrics. For example, we can also decompose *Urban Density* into four factors incorporating a common measure, *Net Residential Density,* defined by the average number of dwelling units in a given unit of residential area. *Net Residential Density* is typically defined in the literature (e.g. Churchman, 1999; Forsyth 2003) as follows:

(1) *Net Residential Density = Total Number of Dwelling Units ÷ Total Area of Residential Plots.*

The reader can ascertain that *Net Residential Density* is indeed a product of four of the factors in Equation (16):

(2) *Dwelling Unit Packing × Floor Plan Efficiency × Building Height × Plot Coverage = Net Residential Density.*

Therefore, replacing these four factors with *Net Residential Density* in Equation (16), we obtain *Urban Density* as a product of four factors:

(3) *Dwelling Unit Occupancy × Occupancy Rate × Net Residential Density × Residential Share = Urban Density.*

11 Alonso (1968) explains that for models involving multiplication (or division), the error for a product (or the ratio) is calculated as follows:

(1) $z = xy$ or $z = x/y$.

(2) $e_z^2 = y^2e_x^2 + x^2e_y^2$, $e_z^2 = e_x^2/y^2 + e_y^2/x^2$ if x and y are not correlated. If they are correlated, then

(3) $e_z^2 = y^2e_x^2 + x^2e_y^2 + xye_xe_yr$, where r is the correlation coefficient.

Since many of the values in our models were derived from other values, they are subject to cumulative errors. This is especially true of our estimates of the total residential floor area in the city, the total occupied floor area, dwelling unit size, or occupied floor area per person. In the Technical Annex we compare our derived values for dwelling unit size with values found in published articles and reports. In most cities we studied, the values are within a reasonable range of each other.

12 We classified the *Landsat* imagery into built-up and non-built up pixels. We then classified the built-up pixels into urban, suburban or rural ones, based on the shares of built-up pixels within a 1km^2 walking distance circle around them: Those with less than 25% were classified as rural; those with 25-50% were classified as suburban; and those with 50% or more were classified as urban. We created urban clusters by grouping contiguous urban and suburban pixels. We included fringe open spaces that were within 100 meters of them and captured open spaces that were fully enclosed by urban and suburban pixels and fringe open spaces and were less than 200 hectares in area in the urban clusters. Urban clusters that shared buffers surrounding them equal to one-quarter of their area were then combined to form the city's *Urban Footprint*. This method for defining urban footprints was adopted by the United Nations for the purpose of monitoring the U.N. Sustainable Development Goals (UN-Habitat, 2019).

[13] Maps of the enumeration districts and population data for districts for these cities were provided to the authors by the Center for International Earth Sciences Information Network (CIESIN) as Columbia University.

[14] Each census enumeration district contains built-up pixels and the population within each enumeration district is assumed to be living within those built-up pixels. It is further assumed that the population is equally divided among those built-up pixels. Only the portion of the population living in pixels that fall within the urban footprint, as defined earlier, is counted in the population of the city.

[15] This classification was based on a taxonomy developed for the *Atlas of Urban Expansion: 2016 Edition* (Angel *et al*, 2016). A point that fell on a plot or a building with mixed land use including residential use was considered to be 'residential.' Residential Share was estimated as the share of 'residential' points in the total number of Halton points sampled.

[16] In the majority of cases it was possible to count the number of floors in high-resolution satellite imagery or *Google Street View* based on window openings and balconies. Occasionally, the number of floors in a building in our sample was estimated by comparing it with an adjacent building of similar height with visible windows, or by comparing the length its shadow with that of an adjacent building with visible windows.

[17] For each architectural floor plan, analysts distinguished living and non-living areas, with areas that are exclusively within private dwelling units categorized as 'living areas.' The average ratio of living area to total floor area was calculated for each building type, and then a weighted average ratio was calculated for each city based on the mix of building types identified in the *Building Height* measurement. In some cities, published reports were consulted as well.

[18] All of these values are ratios rather than totals and these ratios are all 'normalized'—i.e. independent of city totals of one kind or another—and therefore comparable from one city to another.

[19] The authors believe that these factors can and should be used to monitor densification efforts. Although the methods outlined here are robust, monitoring changes in factors over time may require further refinement as well as testing on the ground in several pilot cities.

[20] Several real-world examples of successes in implementing each one of these interventions as well as failures in their application are described detail elsewhere (Angel and Lamson-Hall, 2020).